Empirically Evaluating the Electoral College

Jonathan N. Katz
California Institute of Technology*

Andrew Gelman
Columbia University†

Gary King
Harvard University and
the World Health Organization‡

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The 2000 U.S. presidential election has once again rekindled interest in possible electoral reform. While most of the popular and academic accounts have focused on balloting irregularities in Florida, such as the now infamous "butterfly" ballot and mishandled absentee ballots, some have also noted that this election marked only the fourth time in history that the candidate with a plurality of the popular vote did not also win the Electoral College. This "anti-democratic" outcome has fueled desire for reform or even outright elimination the Electoral College.

This is not the first time that such controversy has surrounded the Electoral College system. Perhaps the most scandalous presidential election in U.S. history was the 1876 election between Samuel J. Tilden, a Democrat, and Rutherford B. Hayes, a Republican. The nation was deeply divided; caused partly by a deep economic recession as well as a seemingly endless number of scandals involving graft and corruption in the incumbent Republican administration of Grant. Making matters more divisive, there were also a number of third parties that contested the election. Similarly to the 2000 elections, the outcome hinged on resolving potential vote count problems in Florida, Louisiana, and South Carolina. These states were so divided, as was the rest of the country, that they sent two slates of electors

*Division of the Humanities and Social Sciences, D.H.S.S. 228-77, Pasadena, CA 91125; jkatz@caltech.edu, http://jkatz.caltech.edu.

†Department of Statistics and Department of Political Science, Columbia University, New York, NY 10027; gelman@stat.columbia.edu, http://www.stat.columbia.edu/~gelman.

‡Center for Basic Research in the Social Sciences, 34 Kirkland Street, Harvard University, Cambridge MA 02138; http://gking.harvard.edu, king@harvard.edu.

each to Congress—one set for Tilden and one set for Hayes. The Congressional procedures for resolving the disputed set of electors had expired, therefore Congress established a 15 member commission to decide the issue of which set of electors to use. After much intrigue, the commission narrowly voted to use the electors for Hayes from all three disputed states thereby giving the election to him. Hayes won the election despite that the consensus was that Tilden had won 51% of the popular vote to Hayes' 48%.

Another case in which the Electoral College vote went contrary to the popular vote was the 1888 election between the incumbent President Grover Cleveland, and his Republican challenger, Benjamin Harrison. Cleveland garnered huge majorities in the 18 states that supported him, whereas Harrison won slender majorities in some the larger states that supported him. In the end, Cleveland won the popular vote by about 110,000 votes. constituting less than one percent, but lost the Electoral College.

The other election in which the popular-vote leader did not become President was in 1824, when Andrew Jackson won a plurality of both the popular and electoral votes. But, because Jackson did not win an electoral-vote majority, the election was decided by the House of Representatives, which voted for John Quincy Adams.

The popular vote was even closer in the 2000 election. Gore won the popular vote by approximately 541,000 votes, or about about half a percentage point of the total vote cast but, as with both Cleveland and Tilden before him, lost the Electoral College.

Most arguments against the Electoral College have either been based on these particular elections or on highly stylized formal models (see, for example, Banzhaf 1968). We take a very different approach here. We develop a set of statistical models based on historical election results to evaluate the Electoral College as it has performed in practice. Thus, while we do not directly address the normative question of the value of the U.S. Electoral College, this paper does provide the necessary tools and evidence to make such an evaluation.

There are two fundamental ways that the Electoral College could potentially be flawed. First, it may be biased in favor of one party. That is, the distribution of votes could have a party's candidate winning the popular vote but losing the Electoral College. For example, if it is likely that the Democratic candidate is to win with overwhelming majorities in a few states, then this will boost the overall Democratic vote share but not necessarily the odds of them winning the Electoral College. This is essentially what happened in the elections of 1876, 1888, and 2000. In order to know if this is a general problem, we need to know what is the relationship between average vote a party's candidate receives and the likelihood of winning a majority of the Electoral College. This is directly analogous to a seats-votes curve used to study legislative elections. We develop a model based on an extension of Gelman and King (1994) to the case of the Electoral College.

The second possible affect of the Electoral College is on the voting power of individuals citizens—that is, their influence on the outcome. A natural measure of voting power is the probability that a vote will be pivotal in determining the outcome of an election. This is the basis for almost all of the voting power measures considered in the literature (see Straffin 1978 and Felsenthal and Machover 1998, for a review). If the president were elected by straight popular vote, this is just probability that your vote breaks a tie in favor of a candidate. Under the popular vote system every voter (*ex ante*) has an equal, but small chance, of casting the deciding vote and therefore has equal voting power. Further, the popular vote system maximizes the average voting power across the electorate (see, e.g., Felsenthal and Machover 1998, and Gelman and Katz 2001). The Electoral College, on the other hand, divides the electorate into predetermined coalitions by state. The states give all of their electoral votes to a given candidate based on majority vote in the state[1]. Thus, a vote is pivotal if it determines how the state's electoral votes are cast and if those electoral votes determine the winner in the Electoral College. Since states vary both their sizes and likelihood of ties, voters in different states will in general have have different voting power and as a result the average voting power can be less than under plurality rule. In fact, this is the central critique raised by Banzhaf (1968). Again using the statistical model of presidential elections we develop, we can examine the empirical probability that an average voter is pivotal under both popular vote and Electoral College systems.

We show that when one preforms appropriate statistical analysis of the available historical elections data, there is not much basis to argue for reforming the Electoral College. We first show that while the Electoral College may once have been biased against the Democrats, given the current distribution of voters, neither party is advantaged by the system. Further, the electoral vote will differ from the popular vote will only when the average votes shares are very close to a half. As for voting power, we show that while there has been much temporal variation in voting power over the last several decades, the voting power of individual citizens would not likely increase under a popular vote system of electing the president.

[1]Actually two states, Nebraska and Maine, allocate Electoral College votes by Congressional district.

The paper proceeds as follows. In the next, section we consider the partisan impact of the Electoral College. Then in Section 3 we examine the impact on average voting power, including what would voting power have been if electoral votes were allocated by congressional district or if a popular vote system had been used. The final section concludes.

2. MEASURING THE PARTISAN IMPACT OF THE ELECTORAL COLLEGE

Most of the popular critiques of the Electoral College have focused on the possibility that the winner of the popular vote will not win the electoral vote, such as occurred in the elections of 1876, 1888, and 2000. That this could happen should not be a surprise because they are different electoral systems. In fact, the Constitutional Convention considered several alternative methods for electing the president, including direct popular vote, and rejected them all in favor of the Electoral College. The convention delegates were concerned that elections would be dictated by the most populous states with little regard for the smaller ones.[2] This was thought particularly likely to occur if one of the presidential candidates was a "favorite son" from one of larger states, who would thus draw large support from only one state or region but would still able to win the popular vote.

If a primary rationale for eliminating the Electoral College is that the results may be contrary to the outcome of the popular vote we need to know how likely this is to happen. We can not answer this question directly (except in the general sense that this has happened four times in ab out 45 elections). However, we can investigate the relationship between popular vote and the probability that a party's candidate wins the electoral college. We can, for example, examine what popular vote share would be needed for the Democratic candidate to win the Electoral College 50 percent of the time, or even 95 percent of the time. If the vote needed to win 50% of the time were substantially greater then half of the popular vote, we would know that the distribution of votes across the states disadvantaged the Democrats. In other words, the Electoral College would be biased in favor of the Republicans in this case.

How might the Electoral College favor one of the parties? Think of the Electoral College as a legislative district map. A central concern about any districting is how it affects the translation of votes into seats. The study of this translation in the political science literature is based on the idea of a seats-votes curve, which has appeared in the academic literature

[2] See also *Federalist Paper, No. 69* for further arguments in favor of the system.

for almost a century (see, for example, Kendall and Stuart 1950). A seats-votes curve is a mapping stating for a given party's average district vote what fraction of the seats will they receive. The Electoral College represents just a slightly more complicated "seats-votes" curve. We are interested in knowing the relationship between average popular vote and electoral vote. The complication is that the districts in our case—that is, the states—have different numbers of seats—that is, electoral votes.

As is well known, a legislative districting map may be a gerrymander in favor of a particular party. The classic way to engineer such a partisan gerrymander is to pack as many of the other party's supporters in as few districts as possible, thereby creating inefficiently safe districts, while spreading its own supporters across as many districts as possible thereby creating winnable but not inefficiently safe districts (see Cox and Katz 1999 & 2002, for detailed decision of this and other methods of partisan gerrymandering). The opposition voters in these districts are wasting their votes: they are contributing to increasing the statewide vote share for their party, but are not increasing the number of seats they win. In the case of legislative maps, this gerrymander is often intentional. With the Electoral College, such gerrymandering is not likely intentional because the allocation is set in the Constitution not by partisan state governments as with legislative maps. Nevertheless, it could be the case that distribution of voters across states could create the conditions for a gerrymander favoring one of the parties in the presidential election. If, for example, many Southern states vote overwhelmingly for the Democrat, then the Democratic candidate may do well in the popular vote but still may not win the Electoral College.

If we look at the 2000 election the distribution of popular vote was problematic for Gore. Consider New York, the second most populous state, where Gore won there with over 60% of the votes cast to Bush's 35%. Many of these Democratic voters in New York in essence wasted their votes. If they could have been transferred to another state, Gore could have easily won the Electoral College. On the other hand, Bush "wasted" a large number of votes in Texas. But California went strongly for Gore . . . Clearly, a systematic approach is needed to go beyond anecdotal reasoning.

In order to know if this is a problem in general, we need to estimate the seats-votes curve. The methodology we will is an extension of a model developed by Gelman and King (1994), where the full details can be found.

The procedure consists of two parts. First, using historical elections results we generate a statistical forecasting model that relates the observable characteristics of a state to the presidential vote. That is, the forecasting model tells us our best estimate for the expected Democratic candidate's vote in a state a given set of characteristics. We also get an estimate of how variable elections are over time.

A bit more formally, our probability model comes from a random components linear regression of Democratic vote in state i in year t, $v_{i,t}$, on a set of observable regressors, $X_{i,t}$,

$$v_{i,t} = X_{i,t}\beta_t + \gamma_i + \varepsilon_{i,t}, \qquad (1)$$

where β_t is a vector of k parameters that must be estimated from the data, and γ_i and $\varepsilon_{i,t}$ are independent error terms. We further assume that both error terms are normally distributed with mean zero and variances that are estimated from the data. This implies that the vote shares themselves are also normally distributed around the expected conditional mean defined in Equation 1.

The variables $X_{i,t}$ we used to forecast the presidential vote are from Campbell's (1992) study. They include each state's deviation from previous average national presidential vote in the last two elections, indicators for presidential and vice-presidential nominees' home states, the partisan breakdown in the lower chamber of the state's legislature, the economic growth rate in the state, and liberalness of state Congressional delegation based on ADA and ACA ratings (see Campbell 1992, for the complete descriptions).

Once we have the forecasting model, we can then consider what would happen as electoral conditions change. For example, we could consider hypothetical partisan swings such that the average vote varied from the actual election results. By letting these swings vary we can map out the seats-votes curve.

The probability model of the hypothetical vote proportions, $v_{i,t}^{hyp}$ is determined by the analogous probability model:

$$v_{i,t}^{hyp} = X_{i,t}^{hyp}\beta_t + \delta_t^{hyp} + \gamma_i + \varepsilon_{i,t}^{hyp},$$

where δ_t^{hyp} is a known constant used to model statewide partisan swings[3].

In our case, we are interested in evaluating actual election results, so we will choose $X_{i,t}^{hyp}$ to equal the original values for the state. However, by appropriately choosing the partisan swing, δ_t^{hyp}, we can see what the vote distribution would be like as the national average vote varies. This will in turn allow us to calculate the probability that a given state will be won by the Democrats for a given statewide vote.

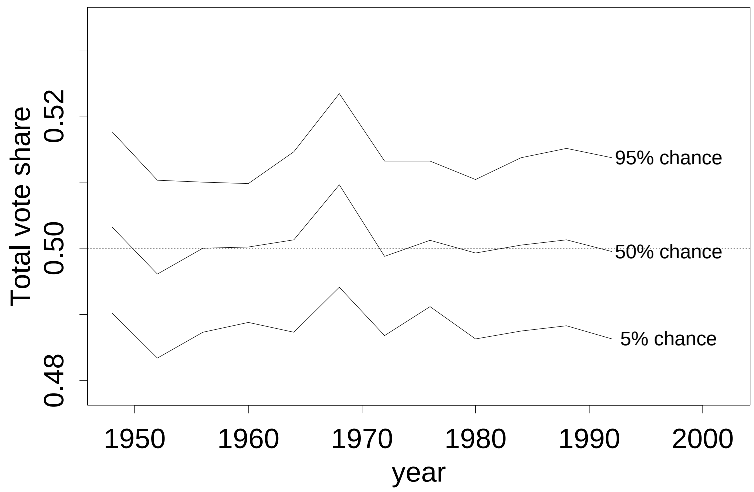

Figure 1: *Democratic share of the two-party vote required for a 5%, 50%, and 95% of wining the Electoral College for each presidential election from 1948 to 1992.*

We can then estimates quantities of interest, such what average vote shares are necessary for the Democrats to have a 50% or 95% chance of winning the Electoral College. These results are presented in Figure 1. As we can see from the figure, if the average vote share is near 50%, the Democrats have about even odds of winning the electoral college. If their average vote share were over 51%, the Democratic candidate would almost surely win. The only election which seems to be different at all is the 1968 election. If conditions were like they were in 1968, the Democrats seem slightly disadvantaged by the Electoral College but

[3]If we want to have the average vote equal to some value, $\bar{v}$, then this constant can be calculated by:

$$\delta_t^{hyp} = \bar{v} - \sum_{i=1}^{N} (X_{i,t}^{hyp} \beta_t) T_{i,t} / \sum_{i=1}^{N} T_{i,t}$$

where N is the number of states and $T_{i,t}$ is the voter turnout in state i in year t.

otherwise it seems relatively fair. Further, as long as one the winner of the popular vote does so by winning on average 51% of the two-party vote, the results from the Electoral College and popular vote will agree.

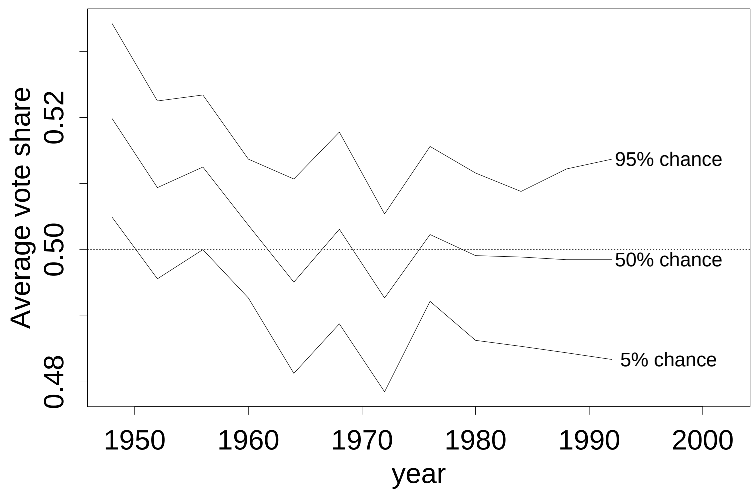

Figure 2: *Democratic share of the two party population-weighted vote required for a 5%, 50%, and 95% of wining the Electoral College for each presidential election from 1948 to 1992*

There is, however, a potential problem with the analysis presented in Figure 1. Implicitly, we are conditioning on turnout because the average vote is just the simple average across the states. However, if there are wide turnout differentials between states, as there are, this may mask some findings. As an alternative, we also ran the analysis constructing the national average by weighting state vote shares according to their actual population. These results appear in Figure 2.

We see some differences between the two figures. Most notably, early on the Electoral College seems somewhat biased against the Democrats, especially during the 1950s. That is, the Democratic candidate needed larger popular vote shares to guarantee high chances of winning the election. This was caused because the Democrats were winning Southern states with huge margins, thy they had lots of "wasted votes." The effect has reduced over time because the Republicans have started to win Southern states. This did not turn up in the previous figure as clearly, because turnout was also much lower in the South counteracting the wasted votes.

Thus, we see some evidence that in the 1940s and 1950s, the Electoral College could have led to results contrary to the popular vote. In current elections, this can only really happen if the popular vote is very close, as it was in the 2000 election. In the 1980s, commentators talked about a Republican "lock" on the Electoral College, but really what was happening was that Republicans were winning Presidential elections by getting many more votes than the Democrats. The Electoral College had nothing to do with it, as is clear from the segments of the graphs in Figures 1 and 2 showing the 1980s.

3. ELECTORAL COLLEGE AND VOTING POWER

In this section, we explore how coalitional behavior induced by the Electoral College affects the probability a given voter is decisive in an election, a natural measure to evaluate an electoral system.

The probability of a vote being decisive is important directly—it represents your influence on the electoral outcome, and this influence is crucial in a democracy—and also indirectly, because it could influence campaigning. For example, one might expect campaign efforts to be proportional to the probability of a vote being decisive, multiplied by the expected number of votes changed per unit of campaign expense, although there are likely strategic complications since both sides are making campaign decisions (c.f. Brams and Davis, 1974). The probability that a single vote is decisive in an election is also relevant in determining the utility of voting, the responsiveness of an electoral system to voter preferences, the efficacy of campaigning efforts, and comparisons of voting power (Riker and Ordeshook 1968, Ferejohn and Fiorina 1974, Brams and Davis 1975, Aldrich 1993).

Perhaps the simplest measure of decisiveness is the (absolute) Banzhaf (1965) index, which is the probability that an individual vote is decisive under the assumption that all voters are deciding their votes independently and at random, with probabilities 0.5 for each of two candidates. We shall refer to this assumption as the *random voting model*. While clearly an unrealistic assumption, it does provide a benchmark to evaluate competing electoral rules and make the problem theoretically tractable. The random voting model is, of course, a gross oversimplification, and in fact its implications for voting power in U.S. elections have been extremely misleading in the political science literature, as has been discussed by Gelman, King, and Boscardin (1998) and Gelman, Katz and Bafumi (2002).

We offer an alternative approach here based on the empirical analysis of U.S. Presidential elections. We use results from every election since 1960 as the basis of a set of simulations to calculate the average probability that a given voter is decisive under the popular vote, the electoral college, and an alternative system in which each Congressional district is worth one electoral vote.

3.1. *How the Electoral College might affect Voting Power*

Before continuing on it is probably best to consider how the Electoral College might affect voting power. In order to to this we need to consider a couple of different electoral systems. The simplest electoral system is the *popular vote* or *majority rule*, under which the candidate receiving the largest number of votes wins. On the other hand, the U.S. Presidential system, is an *electoral vote* or *local winner-take-all* rule, in which the voters are grouped into several coalitions, in which the winner in each coalition gets a fixed number of "electoral votes" (with these electoral votes split or randomly assigned in the event of an exact tie within the coalition), and then the candidate with the most electoral votes is declared the winner.

In a popular vote system with n voters, the probability that your vote is decisive is $\binom{n-1}{(n-1)/2}2^{-(n-1)}$; that is, the probability that $x = \frac{n-1}{2}$ where x has a binomial distribution with parameters $n-1$ and $\frac{1}{2}$. For large (or even moderate) n, this can be well approximated using the normal distribution as $\sqrt{\frac{2}{\pi}}n^{-1/2}$, a standard result in probability (c.f., Woodroofe, 1975).

Now consider *coalitions*, in which the members of a coalition with m members have the prior agreement that they will separately caucus, and then the winner of the vote in their coalition will get all of their m votes. This is just another way of describing the Electoral College.

Under the random voting model, it is smart to join a coalition. At one extreme, suppose a single coalition has $\frac{n+1}{2}$ voters. Then this coalition determines the election outcome, and if you are in that coalition, your vote is decisive with approximate probability $\sqrt{\frac{2}{\pi}}\left(\frac{n}{2}\right)^{-1/2}$, which is approximately $\sqrt{2}$ times the probability of being decisive under the popular vote system. However, the $\frac{n-1}{2}$ voters not in the grand coalition have zero voting power; thus, the *average* probability of a decisive vote, averaging over all voters, is approximately $\sqrt{\frac{1}{\pi}}n^{-1/2}$, a factor of $\sqrt{2}$ *less* that under the popular vote system.

A. No Coalitions.
A voter is decisive if the others are split 4-4:

$$\Pr(\text{Voter is decisive}) = \binom{8}{4} 2^{-8} = 0.273$$

Average Pr(Voter is decisive) = 0.273

B. A Single Coalition of 5 Voters.
A voter in the coalition is decisive if others in the coalition are split 2-2:

$$\Pr(\text{Voter is decisive}) = \binom{4}{2} 2^{-4} = 0.375$$

A voter not in the coalition can never be decisive:

$$\Pr(\text{Voter is decisive}) = 0$$

Average Pr(Voter is decisive) = $\frac{5}{9}(0.375) + \frac{4}{9}(0) = 0.208$

C. A Single Coalition of 3 Voters.
A voter in the coalition is decisive if others in the coalition are split 1-1 and the coalition is decisive:

$$\Pr(\text{Voter is decisive}) = \frac{1}{2} \cdot \frac{50}{64} = 0.391$$

A voter not in the coalition is decisive with probability:

$$\Pr(\text{Voter is decisive}) = \binom{5}{1} 2^{-5} = 0.156$$

Average Pr(Voter is decisive) = $\frac{3}{9}(0.391) + \frac{6}{9}(0.156) = 0.234$

D. Three Coalitions of 3 Voters Each.
A voter is is decisive if others in the coalition are split 1-1 and the other two coalitions are split 1-1:

$$\Pr(\text{Voter is decisive}) = \frac{1}{2} \cdot \frac{1}{2} = 0.250$$

Average Pr(Voter is decisive) = 0.250

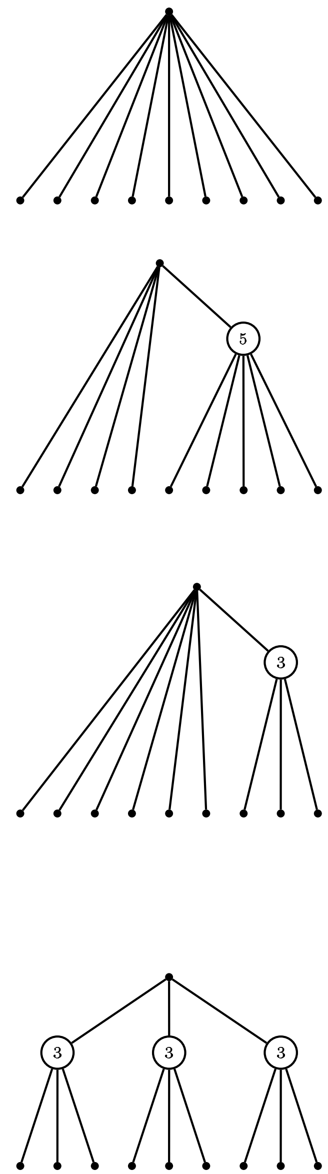

Figure 3: *An example of four different electoral systems with 9 voters. Each is a "one person, one vote" system, but they have different implications for probabilities of casting a decisive vote. The average probability of decisiveness is maximized under A, the popular-vote rule.*

Voters benefit even under small coalitions. For a simple example, consider a election with 9 voters under different electoral rules as depicted in Figure 3. Under the popular vote system, any voter's chance of being decisive is $\binom{8}{4}2^{-8} = 0.273$. Now suppose that 3 voters are in a coalition and the other 6 vote independently. Then how likely is your vote to be decisive?

If you are in the coalition, it is first necessary that the other 2 voters in the coalition be split; this happens with probability 1/2. Next, your coalition's 3 votes are decisive in the entire election, which occurs if the remaining 6 voters are divided 3-3 or 4-2; this has probability $\frac{50}{64}$. The voting power any of the 3 voters in the coalition is then $\frac{1}{2} \cdot \frac{50}{64} = 0.391$. What if you are not in the coalition? Then your vote will be decisive if the remaining votes are split 4-4, which occurs if the 5 unaffiliated voters (other than you) are split 4-1 in the direction opposite to the 3 voters in the coalition. The probability of this happening is $\binom{5}{1}2^{-5} = 0.156$. Compared to the popular vote system, you have more voting power if you are in the coalition and less if you are outside. The average voting power is $\frac{3}{9}0.391 + \frac{6}{9}0.156 = 0.234$, which is lower than under the popular vote system (see Panel A of Figure 3).

Finally, one can consider more elaborate arrangements. For example, suppose there are $n = 3^d$ voters, where d is some integer, who are divided into three equal-sized coalitions, each of which is itself divided into three coalitions, and so forth, in a tree structure. Then all the n voters are symmetrically-situated, and a given voter is decisive if the other 2 voters in his or her local coalition are split—this happens with probability $\frac{1}{2}$—and then the next two local coalitions must have opposite preferences—again, with a probability of $\frac{1}{2}$—and so on up to the top. The probability that all these splits happen, and thus the individual voter is decisive, is $\frac{1}{2^d} = n^{-\log_3 2} = n^{-0.63}$, which is lower than the probability under the popular vote system (for large n, that probability is approximately $0.8n^{-0.5}$). For example, if $n = 3^8 = 6561$, then the probability of a decisive vote is 1/256 with the tree-structure of coalitions, compared to about 1/102 with majority rule.

The above examples indicate that under the random voting model, it is to your benefit to be in coalitions, with larger coalitions generally being better. If you are in a coalition of size m, the probability that your coalition is tied is approximately proportional to $m^{-1/2}$, and the probability that your coalition is itself required to determine the election winner is approximately proportional to m; the product of these two probabilities thus increases with m, at least for $m << n$. A similar logic leads to large coalitions themselves fragmenting into sub-coalitions.

In a system such as the U.S. Electoral College, the coalitions are set ahead of time rather than being subject to negotiation, and the mathematical analysis just presented suggests that voters in larger states have more voting power (Mann and Shapley 1960, Banzhaf 1968, Rabinowitz and Macdonald 1986). This comparison is not valid in practice, however, given the observed departures from the random voting model (see Margolis 1983, and Gelman, Katz, and Bafumi 2002). In fact, voters in the very smallest states tend to have higher voting power, on average (see Gelman, King, and Boscardin 1998).

3.2. *Estimating voting power empirically*

As has been noted by many researchers (e.g., Beck 1975, Margolis 1977, Merrill 1978, and Chamberlain and Rothchild 1981), there are theoretical and practical problems with a model that treat votes as independent coin flips (or, equivalently, that counts all possible arrangements of preferences equally). The simplest model extension is to assume votes are independent but with probability p of voting for one of the candidates, say the Democrat, with some uncertainty about p (for example, p could have a normal distribution with mean 0.50 and standard deviation 0.05). However, this model is still too limited to describe actual electoral systems. In particular, the parameter p must realistically be allowed to vary, and modeling this varying p is no easier than modeling vote outcomes directly. Following Gelman, King, and Boscardin (1998), one might try to construct a hierarchical model, as they did for U.S. Presidential elections with uncertainty at the national, regional, and state levels.

We consider a different approach to modeling whether a single vote is decisive. Consider a two-candidate election with majority rule in any given jurisdiction. Let V be the proportional vote differential (e.g., the difference between the Democrat's and Republican's vote totals, divided by the number of voters, n). If you vote, that will add $+1/n$ or $-1/n$ to V; the decisiveness of this vote is is 0 if $|V| > 1/n$, 1/2 if $|V| = 1/n$, or 1 if $V = 0$.

Now suppose that that the proportional vote differential has an approximate continuous probability distribution, $p(V)$. This distribution can come from a theoretical model of voting (e.g., the random voting model) or empirical models based on election results or forecasts. Gelman, King, and Boscardin (1998) argue that, for modeling voting decisions, it is appropriate to use probabilities from forecasts, since these represent the information available to the voter before the election occurs. For retrospective analysis, it may also be interesting to use models based on perturbations of actual elections as in Gelman and King (1994). In any case, all that is needed here is some probability distribution.

For any reasonably-sized election, we can approximate the distribution $p(V)$ of the proportional vote differential by a continuous function. In that case, the expected probability of decisiveness is simply $2p(V)/n$ evaluated at the point $V = 0$.[4] For example, in a two-candidate election with 10,000 voters, if one candidate is forecast to get 54% of the vote with a standard error of 3%, then the vote differential is forecast at 8% with a standard error of 6%. The probability that an individual vote is decisive is then $2\frac{1}{\sqrt{2\pi}(0.06)}\exp(-\frac{1}{2}(0.08/0.06)^2)/10000 = 0.0055$, using the statistical formula for the normal distribution.

The same ideas apply for more complicated elections, such as multicandidate contests, runoffs, and multistage systems (e.g., the Electoral College in the U.S. or the British parliamentary system in which the goal is to win a majority of individually-elected seats). In more complicated elections, it is simply necessary to specify a probability model for the entire range of possible outcomes, and then work out the probability of the requisite combination events under which a vote is decisive. For example, in the Electoral College, your vote is decisive if your state is tied (or within one vote of tied) and if, *conditional on your state being tied*, no candidate has a majority based solely on the other states. Estimating the probability of this event requires a model for the joint distribution of the vote outcomes in all the states.

Therefore, in order to estimate probabilities of close elections and decisiveness, it is necessary to set up a probability model for vote outcomes. We want to go beyond the random voting model and set up a more realistic descriptor of vote outcomes. Gelman, King, and Boscardin (1998) fit a state-by-state election forecasting model, with probabilities corresponding to the predictive uncertainty two months before the election. Here, we use a simpler approach: we take the actual election outcome and perturb it, to represent possible alternative outcomes.

We label v_i as the observed outcome (the Democratic candidate's share of the two-party vote) in Congressional district i in a given election year and obtain a probability distribution of hypothetical election outcomes y_i by adding normally-distributed random errors at the national, regional, state, and Congressional-district levels, with a standard deviation of 2% at each level. We label n_i as the turnout in each district i and consider these as fixed—this is reasonable since uncertainty about election outcomes is driven by uncertainty about v, not n.

For any given election year, we use the multivariate normal distribution of the vector v of vote outcomes to compute the probability of a single vote being decisive in the election. For

[4]If the number of voters n is odd, this approximates $\Pr(V = 0)$, and if n is even, it approximates $\frac{1}{2}\Pr(V = -1/n) + \frac{1}{2}\Pr(V = 1/n)$.

the popular vote system, we determine this probability for any voter; for the electoral-vote and congressional-district-vote systems, we determine the probability within each state or district and then compute national average probabilities, weighing by turnouts within states or districts. The actual probability calculations are done using the multivariate normal distribution as described by Gelman, King, and Boscardin (1998).

Our results appear in Figure 4. The most striking feature of the figure is that the average probability of decisiveness changes dramatically from year to year but is virtually unaffected by changes in the electoral system. This may come as a surprise—given the theoretical results from Section 3.1, one might expect the average probability of decisiveness to be much higher for the popular-vote system.

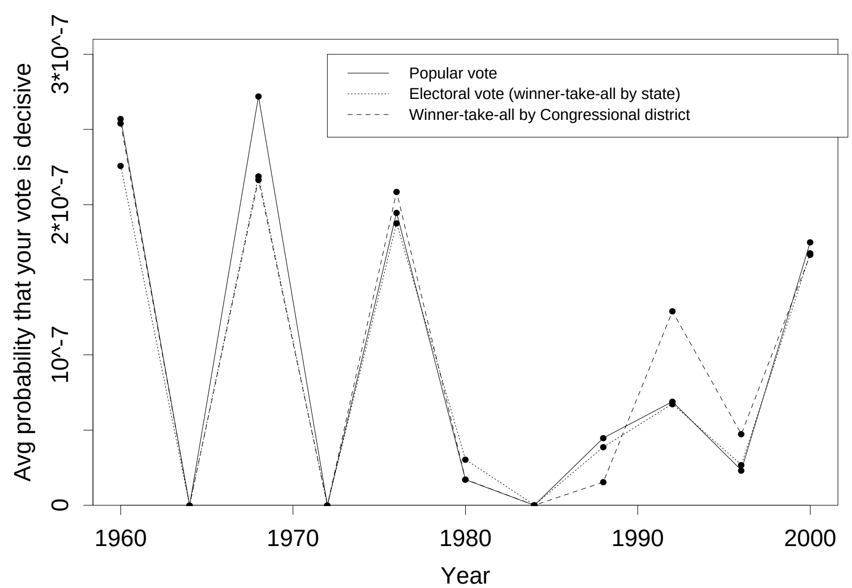

Figure 4: *The estimated average probability of a decisive vote, for each presidential election since 1960, calculated under the popular and electoral vote systems. The estimates are based on simulations from actual election outcomes. The average probability varies a lot by year but is not much affected by the electoral rule.*

The results in Figure 4 are only approximate, not just because of the specific modeling choices made, but also because of the implicit assumption that the patterns of voting would not be affected by changes in the electoral system. For example, states such as California and Texas that were not close in the 2000 election might have had higher turnout under a popular vote system in which all votes counted equally. Thus, our results compare different electoral systems as applied to the actual observed votes and do not directly address counterfactual questions about what would happen if the electoral system were changed.

4. CONCLUSIONS

We have presented statistical methodology to evaluate the two most common complaints against the Electoral College. When subjected to proper empirical analysis, neither of the complaints seems well justified.

With regard to the potential partisan bias in the Electoral College we find no systematic effect, at least not in current elections. We did find that it is possible for the Electoral College system to lead to different outcomes than the popular vote, however only when the nationwide vote is very close for the two top candidates. As we now know from the election of 2002, when elections are that close slight difference is electoral procedure can and will lead to different outcomes.

Our results about voting power and the Electoral College are perhaps a bit more surprising. There is a fairly significant theoretical literature suggesting that the Electoral College was unfair to certain voters. However, these claims are based on the random voting model, which is a highly stylized model of elections. When we looked at the empirical voting behavior, we do not find much difference between voting power under the Electoral College or popular vote systems. But, these results are more tentative because in making our analysis we assumed that neither voters nor candidates would have behaved differently under a popular vote system. This is a suspect assumption, but less so than the random voting model that underlies the critique.

Of course, our empirical findings do not directly address normative questions, such as which electoral system should be used. However, our findings can be used to evaluate the positive claims that underly the normative arguments.

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