1. Reading:


2. Homework due 5pm Tues of week 5:

Consider the model from the previous homeworks, described in Section 3.7 of *Bayesian Data Analysis*:

\[ y_j \sim \text{Binomial}(n_j, \text{logit}^{-1}(\alpha + \beta x_j)), \quad \text{for } j = 1, \ldots, 4, \]

where \( \text{logit}^{-1}(z) = e^z/(1 + e^z) \) and the data are \( x = (-0.86, -0.30, -0.05, 0.73), \) \( n = (5, 5, 5, 5), \) and \( y = (0, 1, 3, 5). \) Assume weakly informative independent \( \text{N}(0, 10^2) \) prior distributions for \( \alpha \) and \( \beta. \) (For Homework 4, I asked for \( \text{N}(0, 100^2) \) priors; this time make the \( \text{N}(0, 10^2). \))

As before, the data represent four experiments of rats exposed to a toxin: \( x_j \) is the logarithm of the dose in experiment \( j, n_j \) is the number of rats in the experiment, and \( y_j \) is the number that die. Thus, in these data, more rats die at higher doses.

The primary quantities of interest in the analysis are \( \beta \) (the slope of the logistic regression), and \(-\alpha/\beta, \) which is called the LD50, the dose at which there is a 50% chance of death.

(a) Use \texttt{optim()} in R to find the posterior mode, the \((\hat{\alpha}, \hat{\beta})\) that maximizes the posterior density. (As always, work with the log-posterior for computational stability.) Call this point \( \hat{\theta} = (\hat{\alpha}, \hat{\beta}). \)

(b) Program a Metropolis-Hastings algorithm with a two-dimensional jumping rule that moves a distance \( \delta \) in the direction of the mode: \( \theta^* | \theta^t \sim \text{N}(\theta^t + \delta \frac{\theta^t - \theta^*}{\|\theta^t - \theta^*\|}, \Sigma). \) This algorithm has two tuning parameters, a scalar \( \delta \) and a \( 2 \times 2 \) covariance matrix \( \Sigma. \)

Choose some simple values: \( \delta = 1, \Sigma = I, \) run the algorithm to approximate convergence, and make sure that the inferences make sense. (You can compare them to your results from Homeworks 3 and 4.)

(c) Put (b) into a R function so it can be run automatically, then do your best to optimize \( \delta \) and \( \Sigma. \) Your goal should be to optimize the efficiency of your Metropolis-Hastings algorithm, which is essentially equivalent in this case to minimizing the autocorrelations of your chains.

Report your best \( \delta \) and \( \Sigma. \) Whoever gets the best results wins a big prize. (Work on your own for this assignment!)