1. Reading:


2. Homework due 5pm Tues of week 4:

Consider the model from the previous homework, described in Section 3.7 of *Bayesian Data Analysis*:

\[ y_j \sim \text{Binomial}(n_j, \logit^{-1}(\alpha + \beta x_j)), \text{ for } j = 1, \ldots, 4, \]

where \( \logit^{-1}(z) = e^z/(1 + e^z) \) and the data are \( x = (-0.86, -0.30, -0.05, 0.73) \), \( n = (5, 5, 5, 5) \), and \( y = (0, 1, 3, 5) \). Assume a noninformative independent \( N(0, 100^2) \) prior distributions for \( \alpha \) and \( \beta \).

The data here represent four experiments of rats exposed to a toxin: \( x \) is the logarithm of the dose in experiment \( j \), \( n \) is the number of rats in the experiment, and \( y \) is the number that die. Thus, in these data, more rats die at higher doses.

The joint posterior density is

\[
p(\alpha, \beta|y, n, y) \propto N(\alpha|0, 100^2)N(\beta|0, 100^2) \prod_{j=1}^{4} [\logit^{-1}(\alpha + \beta x_j)]^{y_j} [1 - \logit^{-1}(\alpha + \beta x_j)]^{n_j-y_j}.
\]

The primary quantities of interest in the analysis are \( \beta \) (the slope of the logistic regression), and \(-\alpha/\beta\), which is called the LD50, the dose at which there is a 50% chance of death.

(a) Program a Metropolis algorithm with a simple two-dimensional jumping rule, \( \theta^* \sim N(\theta, I) \). Run the algorithm to approximate convergence.

(b) Program a Metropolis algorithm with a two-dimensional jumping rule that is optimized for efficiency. Demonstrate that you have set the tuning parameters of your algorithm to be approximately optimal.