1. Reading:


2. Homework due 5pm Tues of week 2:

(a) Program a Gibbs sampler for the linear regression model, given data \((x, y)_i, i = 1, \ldots, n:\)

\[ y_i \sim N(a + bx_i, \sigma^2), \text{ independently, for } i = 1, \ldots, n. \]

with \(a, b\) having independent \(N(0, 10^2)\) prior distributions and \(\sigma = 0.5\).

The only unknowns in the model are \(a\) and \(b\). To implement the Gibbs sampler, first work out \(p(a|b, x, y)\) and \(p(b|a, x, y)\).

To figure these out, start by writing the joint posterior distribution, which is \(p(a, b | x, y) \propto p(a, b) \prod_{i=1}^{n} p(y_i | a, b, x_i)\). Now consider this as a function of \(a\) and \(b\). It has the form, \(p(a, b | x, y) \propto \text{constant} \cdot \exp(\text{quadratic function of } a \text{ and } b)\). (Recall that anything that doesn’t depend on \(a\) or \(b\) is considered a constant in this expression.) Then work out \(p(a|b, x, y)\) by looking at this function considering \(b\) as a constant. When you collect terms you get a normal distribution for \(a\). You can then turn around and do the same thing to get the normal distribution for \(b\) given \(a, x, y\). Equations (2.11) and (2.12) of *Bayesian Data Analysis* might be of help.

Once you have the full conditionals, \(p(a|b, x, y)\) and \(p(b|a, x, y)\), program these, put them in a loop, and run this Gibbs sampler on the data \(x = (1, 2, 3, 4, 5, 6, 7, 8, 9, 10), y = (1, 1, 1, 0, 1, 1, 0, 0, 0, 0)\).

(b) Program a Gibbs sampler for the following probit regression model:

\[ Pr(y_i = 1) = \Phi(a + bx_i), \text{ independently, for } i = 1, \ldots, n, \]

where \(\Phi\) is the cumulative normal distribution function and \(a, b\) have independent \(N(0, 10^2)\) prior distributions.

You can implement Gibbs using continuous latent variables \(z_i\):

\[ z_i \sim N(a + bx_i, 1) \]

\[ y_i = \begin{cases} 1 & \text{if } z_i > 0 \\ 0 & \text{otherwise} \end{cases} \]

The unknowns in the model are \(a, b, z\). To implement the Gibbs sampler, first work out \(p(a|b, z, x, y)\), \(p(b|a, z, x, y)\), and \(p(z|a, b, x, y)\). Then program these and fit to the data \(x = (1, 2, 3, 4, 5, 6, 7, 8, 9, 10), y = (1, 1, 1, 0, 1, 1, 0, 0, 0, 0)\).
Graph and discuss your results. Compare to the results from the following least-squares estimate of the probit regression in R:

```r
library("arm")
M2 <- glm(y ~ x, family=binomial(link="probit"))
display(M2)
```