Working Memory Impairments in Schizophrenia Patients: A Bayesian IRT Analysis

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Background

- **Schizophrenia** - a psychiatric disorder characterized by delusions, hallucinations, and thought disorder.

- **Working Memory** - short term memory. An active system for temporarily storing and manipulating information needed in the execution of complex cognitive tasks (e.g. learning and reasoning).

- **Spatial** Working Memory has been shown to be impaired in schizophrenics
  1. Empirical Evidence: Eye-tracking, hand movement studies
  2. Physical Evidence: Primate studies

- **Object** Working Memory has not previously been studied in schizophrenics.
**Method**

Goal: Compare object and spatial working memory in schizophrenics

1. Is object working memory impaired in schizophrenia patients?
2. If so, is the degree of impairment the same on spatial and object working memory?

- 28 schizophrenics and 33 normal controls participated in memory test with 64 object working memory and 64 spatial working memory items.

- For each test item, participants were first shown two irregular four-sided target polygons on a computer screen. The screen then went blank for three seconds, after which a single probe polygon appeared. Subjects were instructed to answer whether the figure was the same shape (object items) or in the same location (spatial items) as either of the target polygons.
Test Schematic

Subject Fixates

500 ms Targets

3,000 ms Retention

Spatial: Is it in the same location as either target? (correct = No)

Object: Is it the same shape as either target? (correct = Yes)
1. How can we determine whether schizophrenics are more impaired on spatial or on object working memory when both groups did better on spatial tasks?

2. Not all items had equal difficulty, even within object or spatial tasks. Comparing only percent correct data over all trials ignores available information.
Item Response Theory

Item response theory (IRT) analyzes test-takers’ abilities while also analyzing and taking into account properties of the test items themselves.

- Allows abilities to be compared even when test items have differing levels of difficulty.

Main assumption: the probability of answering each item correctly is a monotonic function of subjects’ latent ability $\theta$.

- The exact function (Item Response Function) is determined by latent properties of the item $\beta$.

Item Response Functions generally take the form of CDFs of random variables.
Dichotomous Item Response Model

- $X_{ij}$ represents response of $i$th subject to $j$th test item: $X_{ij} = 1$ if $i$th item was answered correctly and 0 otherwise.
- $\theta_i$ represents ability parameter of $i$th subject
- $\beta_j$ represents characteristics of $j$th item
- Assume $X$ depends only on $\theta$ and $\beta$
- Assume independence of measures within and across subjects, given $\theta$
- Model estimates probability of a correct response:
  
  \[
p_{ij}(\theta_i, \beta_j) = P(X_{ij} = 1|\theta_i, \beta_j)
  \]
Examples

Two-parameter Logistic Model (2PL)

- \( p_{ij}(\theta_i, \beta_j) = \frac{1}{1 + \exp[-\beta_2 \theta_i + \beta_1]} \)

- \( \beta_1 \) represents difficulty and \( \beta_2 \) the discrimination power of \( j \)th item

Three-parameter Logistic Model (3PL)

- \( p_{ij}(\theta_i, \beta_j) = \beta_3 + \frac{1 - \beta_3}{1 + \exp[-\beta_2 \theta_i + \beta_1]} \)

- \( \beta_3 \) represents pseudo-guessing parameter, the probability the \( j \)th item is guessed correctly.
Bayesian IRT Model: Likelihood

- Normals:
  \[ X_{ij}^N \sim \text{Bern} \left( 0.5 + \frac{0.5}{1 + \exp[-\beta_{2j} \theta_{i,t}^N + \beta_{1j}]} \right) \]
  \[ i = 1, \ldots, 33, \quad j = 1, \ldots, 128 \]

- Schizophrenia Patients:
  \[ X_{ij}^S \sim \text{Bern} \left( 0.5 + \frac{0.5}{1 + \exp[-\beta_{2j} \theta_{i,t}^S + \beta_{1j}]} \right) \]
  \[ i = 1, \ldots, 28, \quad j = 1, \ldots, 128 \]

- \( t = 1 \) for object items and \( t = 2 \) for spatial items
Bayesian IRT Model: Prior Distribution

- **Item Parameters**

  Object Items \((k = 1, 2, 3)\):
  \[
  \beta_{1j}^{O,k} \sim N(\eta_1^{O,k}, \tau_1^{O,k}) \quad \beta_{2j}^{O,k} \sim N(\eta_2^{O,k}, \tau_2^{O,k}) : \beta_{2j}^{O,k} > 0
  \]

  Spatial Items \((k = 1, 2, 3)\):
  \[
  \beta_{1j}^{S,k} \sim N(\eta_1^{S,k}, \tau_1^{S,k}) \quad \beta_{2j}^{S,k} \sim N(\eta_2^{S,k}, \tau_2^{S,k}) : \beta_{2j}^{S,k} > 0
  \]

- **Ability Parameters**

  \[
  \theta_i^N \sim N_2(\mu_N, \Sigma_N) \quad i = 1, \ldots, 33 \quad \text{(Normals)}
  \]

  \[
  \theta_i^S \sim N_2(\mu_S, \Sigma_S) \quad i = 1, \ldots, 28 \quad \text{(Schizophrenia Patients)}
  \]

- \(\mu_N[1] = \mu_N[2] = 0\) and \(\Sigma_N[1, 1] = \Sigma_N[2, 2] = 1\) to anchor the model.

- All other parameters have diffuse prior distributions
Comparison with Likelihood Methods

- IRT models pose problems for standard maximum likelihood analysis:
  1. Joint ML estimates of item and ability parameters are not consistent
  2. Marginal ML estimates of item parameters can be calculated using the EM algorithm. Standard errors for ability parameters are then underestimated.

- Analyzing the data in the Bayesian framework automatically takes uncertainty about all other parameters into account, thereby avoiding the problems that arise with maximum likelihood estimates (Patz and Junker, 1999).

- Bayesian inference is less sensitive than likelihood methods to small sample sizes.
## Results

<table>
<thead>
<tr>
<th>Variable</th>
<th>Mean</th>
<th>SE</th>
<th>2.5%</th>
<th>97.5%</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu_S[1]$</td>
<td>-1.10</td>
<td>0.409</td>
<td>-1.99</td>
<td>-0.382</td>
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<tr>
<td>$\mu_S[2]$</td>
<td>-1.17</td>
<td>0.416</td>
<td>-2.03</td>
<td>-0.397</td>
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<td>$\mu_S[1] - \mu_S[2]$</td>
<td>0.070</td>
<td>0.514</td>
<td>-0.948</td>
<td>1.084</td>
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<td>$\Sigma_S[1, 1]$</td>
<td>1.25</td>
<td>0.676</td>
<td>0.371</td>
<td>2.93</td>
</tr>
<tr>
<td>$\Sigma_S[2, 2]$</td>
<td>2.55</td>
<td>1.090</td>
<td>0.980</td>
<td>5.13</td>
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<tr>
<td>$\rho_N$</td>
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<td>0.296</td>
<td>-0.466</td>
<td>0.655</td>
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<tr>
<td>$\rho_S$</td>
<td>0.47</td>
<td>0.271</td>
<td>-0.154</td>
<td>0.896</td>
</tr>
</tbody>
</table>
In addition to spatial working memory, object working memory also appears to be impaired in schizophrenia patients.

– The two domains appear to exhibit the same mean degree of impairment.

– Could be the result of impaired rule-following strategies in schizophrenia patients.

Some evidence of difference between variances of object and spatial working memory.

– Possible indication that schizophrenia does not affect the two domains equally.
Extensions and Future Work

- Include clinically unaffected first-degree relatives of schizophrenia patients to explore models for the genetic inheritance of schizophrenia.
- Try to replicate results found here in different testing situations.
- Further comparisons of likelihood and Bayesian methods for parameter estimation in IRT models.
Comparison of ML and Bayesian Methods for IRT Models:

Simulation Study

- Generate parameters and data sets according to the (2PL) model. Sample size: $I = 35$, $J = 15$. For a fixed set of parameters, generate 500 data sets and analyze each data set using both Bayesian and EM/MMLE methods.

- For each parameter, record the mean estimate and the average coverage rate for 95% intervals.

- Bayesian estimates are based on 5000 draws from the posterior distribution of the parameters after a burn-in of 1500 draws starting from the true parameter values.
Results: Ability Parameters

Mean coverage = .93

Bayesian Coverage Rates for Abilities

Bayesian Mean vs. True Abilities
Interval Coverage: Difficulty Parameters

Mean coverage = .973

Bayesian Coverage Rates for Difficulties

Mean coverage = .957

EM Coverage Rates for Difficulties

Mean coverage = .957
Interval Coverage: Discrimination Parameters

Mean coverage = .954

Bayesian Coverage Rates for Discrimination

Mean coverage = .897
Bias: Difficulty Parameters

Bayesian Mean vs. True Difficulties

EM Mean vs. True Difficulties
Bias: Discrimination Parameters

Bayesian Mean vs. True Discrimination

EM Mean vs. True Discrimination
Conclusions and Future Work

- Bayesian intervals give better coverage
- Bayesian methods perform well with small sample sizes
- For small values of the discrimination parameter, Bayesian estimates appear positively biased.
- Repeat simulations with different sample sizes, parameter values
- How do the parameter estimation methods hold up to violations of model assumptions?
  - Non-normally distributed abilities
  - Non-independent responses
- Carry out similar simulations for Bivariate IRT models.