A Unified View of Hedging and Risk Management

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Abstract

Suppose we are given a semimartingale $S_t$ over a finite time period $[0, T]$, a class of probability measures (or scenarios) $\Lambda$, and a function $f : \Lambda \to \mathbb{R}$. For any $x \in \mathbb{R}$, we say that $x$ can be traded to acceptability, if there exists a suitable $\pi$ satisfying

$$
\mathbb{E}^Q \left[ x + \int_0^T \pi_u dS_u \right] \geq f(Q), \quad \forall Q \in \Lambda.
$$

The set of $x$ which can be traded to acceptability is a half line in $\mathbb{R}$. We show that under some regularity conditions on $\Lambda$ and $f$, the infimum of this set is $\sup f(Q)$, where the supremum is over all probability measures in $\Lambda$ under which $S$ is a martingale.

The above formulation provides us with a unified way of dealing with both hedging and risk management. The minimum value of $x$ can then be interpreted as the minimum capital required so that by careful trading one can avoid risk at the end of the trading period. Another example is an alternative derivation of the superhedging price in incomplete markets.