# What Difference Does the Tiebreak Make in a Tennis Game

## What is special about US Open in contrast to other Grand Slams

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## 1 Introduction

Tennis is a game between two players (or two pairs of players) with a racket and a ball. The match is won by the first player who wins 3 sets (or 2 sets depending on rules). Each particular set consists of games, and it is won when the game score reaches 6:0, 6:1, 6:2, 6:3, 6:4 or 7:5. Players alternate service after each game. The winner of a single game within the set is the player who is the first to win at least 2 more rallies than his/her opponent, under the condition that minimal number of rallies won is 4.

Official tennis rules permit two different approaches when the set score reaches 6:6. One possibility is to continue to play until the first player gets two more games than his/her opponent (Set with no Tiebreak). Second and more popular option is to play a special game, so called tiebreak, to decide the outcome of the set. Players alternate on serve during the tiebreak: the player to serve starts with one serve and after that each player serves twice until the tiebreak is over. Tiebreak is won by the player who is the first to win at least 2 more rallies than his/her opponent, this time the minimal number of rallies won is 7. This situation is called Set with Tiebreak.

Most official tournaments use the tiebreak option to decide outcome of sets when the score reaches 6:6. However, some of them use set with no tiebreak for the last and decisive set (the fifth set in matches best of 5, the third set in matches best of 3). For example, while Grand Slam tournaments Wimbledon, Australia Open and French Open play with no tiebreak in the decisive set, US Open tournament plays all sets with tiebreak. The difference between US Open and other Grand Slam tournaments is studied in this paper.

## 2 Model Specification

One can model the evolution of the tennis score as Markov chain. We can assume that the first player has probability $p$ of winning a rally when he/she serves, while the other player has probability $q$ of winning a rally on his/her serve. All match evolutions eventually lead to win or loss, which could be considered as recurrent (absorbing) states of the Markov chain, while all other intermediate scores represent transient states. General theory of stochastic processes gives methods for computing probabilities of being absorbed in one of the recurrent states, and we use those techniques to determine them. For details, we refer the reader to any book on Markov chains (for instance Lawler [1]).

There are 15 equivalent possible scores representing transient states of the game score, namely 0:0, 15:0, 0:15, 30:0, 15:15, 0:30, 40:0, 30:15, 15:30, 0:40, 40:15, 30:30, 15:40, 40:30, 30:40. The game scores don't count exactly the number of rallies won, so for instance score 15 points represents one rally won, 30 points means 2 rallies won and 40 means 3 rallies won. Situations Advantage In (player on serve is leading by 1 rally and has reached at least 4 won rallies), Deuce (both players have same number of rallies won, but more than 3) and Advantage Out (player on receive is leading by 1 rally and has reached at least 4 won rallies) are equivalent to scores 40:30, 30:30 and 30:40 respectively.

One can set the transition matrix between these scores and use the standard methods for computing absorbtion probabilities to get the desired result. Without going into details, it turns out that probability of winning a single game when it starts (score 0:0) is equal to

$$\frac{p^4\,\left(15-34\,p+28\,p^2-8\,p^3\right)}{1-2\,p+2\,p^2}, \tag{2.1}$$

assuming the player on serve has probability $p$ of winning a single rally.

The situation is much more complex for tiebreak analysis, since there are 52 possible outcomes for the score in the tie-break. Namely: 0:0, 1:0, 0:1, 2:0, 1:1, 0:2, 3:0, 2:1, 1:2, 0:3, 4:0, 3:1, 2:2, 1:3, 0:4, 5:0, 4:1, 3:2, 2:3, 1:4, 0:5, 6:0, 5:1, 4:2, 3:3, 2:4, 1:5, 0:6, 6:1, 5:2, 4:3, 3:4, 2:5, 1:6, 6:2, 5:3, 4:4, 3:5, 2:6, 6:3, 5:4, 4:5, 3:6, 6:4, 5:5, 4:6, 6:5 (identical to “advantage in and serve” scores like 8:7, 10:9, etc.), 5:6 (identical to “advantage out and serve” scores like 7:8, 9:10, etc.), 6:6 (identical to “deuce with serve” scores like 8:8, 10:10, etc.), 7:6 (identical to “advantage in and receive” scores like 9:8, 11:10, etc.), 6:7 (identical to “advantage out and receive” scores like 8:9, 10:11, etc.) and 7:7 (identical to “deuce and receive” scores like 9:9, 11:11, etc.).

It is still possible to obtain closed form solution for winning the tiebreak from any transient score. When tiebreak starts (score is 0:0), the probability of win turns out to be

$$\begin{aligned}\frac{p\,(-1+q)}{-q+p(-1+2q)}\Big(&-((-1+q)^5\,(1+5\,q))\\ &+15\,p\,(-1+q)^4\,q\,(1+5\,q)\\ &-5\,p^2\,(-1+q)^3\,q\,(-11+q+70\,q^2)\\ &+5\,p^3\,(-1+q)^2\,q\,(17-69\,q-28\,q^2+140\,q^3)\\ &-5\,p^6\,q\,(1-14\,q+56\,q^2-84\,q^3+42\,q^4)\\ &+3\,p^4\,q\,(-23+202\,q-473\,q^2+252\,q^3+252\,q^4-210\,q^5)\\ &+p^5\,q\,(29-331\,q+1064\,q^2-1176\,q^3+210\,q^4+210\,q^5)\Big)\end{aligned} \tag{2.2}$$

In order to compare the two options of Set with Tiebreak and Set without Tiebreak, we are left to determine the winning chances in the situation of set without tiebreak. Assuming score 6:6 in the decisive set, the winning chances of the entire match coincide with winning chances in tiebreak which is given by the formula (2.2) above. If there is no tiebreak, the match continues until one player gets two more games than the opponent. This could be simply modelled as Markov chain with just 3 transient states: 6:6, 7:6, 6:7. All other transient scores are equivalent to these three. Since we know probability of winning a game on serve by each player, which is given by formula (2.1), it is then simple to compute match winning chances. The probability of winning the match from score 6:6 without tiebreak is then

$$\frac{\frac{p^4\left(15-34\,p+28\,p^2-8\,p^3\right)}{1-2\,p+2\,p^2}-\frac{p^4\left(15-34\,p+28\,p^2-8\,p^3\right)q^4\left(15-34\,q+28\,q^2-8\,q^3\right)}{(1-2\,p+2\,p^2)\,(1-2\,q+2\,q^2)}}{\frac{p^4\left(15-34\,p+28\,p^2-8\,p^3\right)}{1-2\,p+2\,p^2}+\frac{q^4\left(15-34\,q+28\,q^2-8\,q^3\right)}{1-2\,q+2\,q^2}-\frac{2\,p^4\left(15-34\,p+28\,p^2-8\,p^3\right)q^4\left(15-34\,q+28\,q^2-8\,q^3\right)}{(1-2\,p+2\,p^2)\,(1-2\,q+2\,q^2)}}. \tag{2.3}$$

## 3 Comparison and Most Extreme Differences

### 3.1 Score 6:6 in the Decisive Set

Given that we have closed form solutions for winning probabilities under both options of Set with Tiebreak and Set without Tiebreak, we can analyze the differences. Figure 1 shows the difference between probability of winning in option Set with Tiebreak and in option Set without Tiebreak as a function of probability of winning a single rally on each player's serve. We assume that player on serve has probability $p$ of winning a single rally when he/she serves (plotted as x-axis), while the other player has probability $q$ of winning a rally on his/her serve (plotted as y-axis).

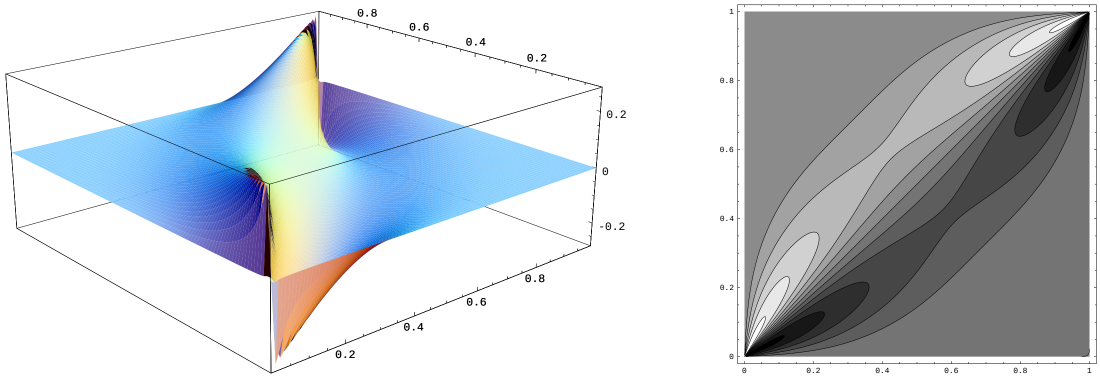

Figure 1: Difference of winning chances in Set with Tiebreak and Set without Tiebreak (3D plot and contour plot) when the score is 6:6 in the decisive set.

One can prove that playing tiebreak is always beneficial for the less consistent player. Therefore the difference between winning chances with tiebreak and without tiebreak are positive above the line $y = x$. The most extreme difference between winning chances happen when either both players are very consistent on their serve ($p \sim 1$, $q \sim 1$), or when they are both very inconsistent ($p \sim 0$, $q \sim 0$). The winning chances can improve as much as 27.45% for the less consistent player if the tiebreak is played over the situation when there is no tiebreak. The difference for more typical serve consistencies $p$ and $q$ is much smaller, but not completely negligible.

Let us study the question how much can the less consistent player improve his/her chances by playing tiebreak in contrast to no tiebreak situation. Figure 2 representing this case is plotted below on the left. The x-axis is the probability of winning a rally on serve by the less consistent player. The red curve is the maximal increase in winning chances by the less consistent player if the match is decided in tiebreak in contrast to no tiebreak. The match winning chances corresponding to this maximum are plotted in blue (with tiebreak) and in green (without tiebreak). The graph on the right is the difference between serve probabilities of the more consistent and the less consistent player, which achieve this maximum (x-axis is the probability of winning a rally on serve by the less consistent player).

As seen from the graphs, the maximum is often achieved when the consistency of the better of the two players is relatively high, meaning that the less consistent player has small chances to win the match with or without the tiebreak. On the other hand, tiebreak can improve the chances of the less consistent player by at least 10% in the extreme cases.

### 3.2 Score 0:0 in the Decisive Set

We can generalize the comparison analysis between tiebreak and no tiebreak situations to include cases of any initial tennis score. Let us consider the case when the decisive set is at the very beginning, i.e., the score

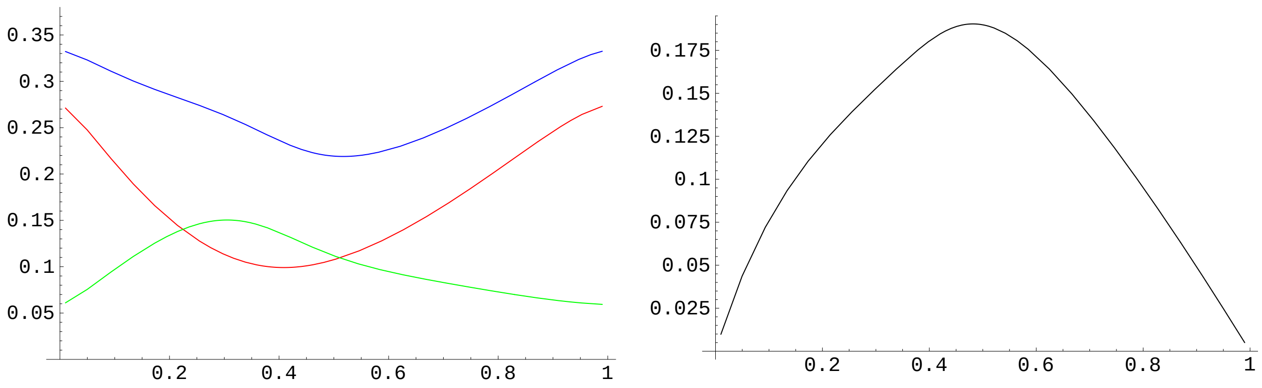


Figure 2: On left: the maximal difference of winning chances (score 6:6 in the decisive set) as a function of probability of winning on serve by the less consistent player. Red curve is the difference, blue curve the corresponding chances of winning the match with tiebreak, green curve without tiebreak. On right: the difference between winning the rally on serve between the more consistent player and the less consistent player which achieves the maximum in the left graph as a function of probability of winning on serve by the less consistent player.

is 0:0 in the last set. Again, by setting the corresponding Markov chain model, we can compute the difference of winning chances in the two cases of tiebreak and no tiebreak.

As one would expect, many score scenarios will result in win and loss without ever going into the situation of getting 6:6 score. Therefore the difference between tiebreak and no tiebreak is less pronounced in score 0:0 in the last set in contrast to score 6:6 in the last set.

The most interesting analysis is again the search for most extreme situations. Figure 3 shows the maximal difference between winning chances in Set with tiebreak in contrast to Set without tiebreak as a function of consistency on serve of the weaker player. The weaker player can improve his/her chances by at most $\sim 1\%$ if his/her consistency on serve is between 35% and 55%. The maximal difference quickly increases outside that region, but it is still much smaller in comparison to score 6:6.

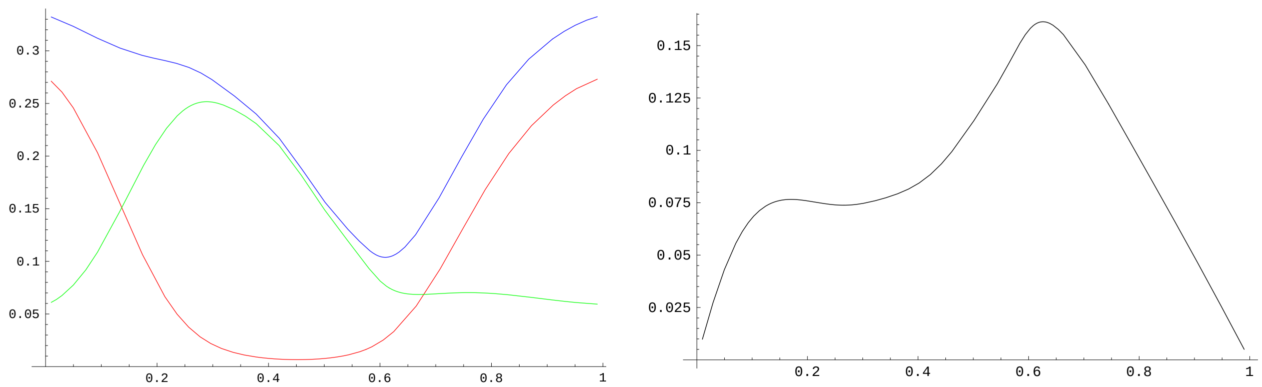


Figure 3: The same graph as Figure 2 representing situation of score 0:0 in the decisive set.

## 3.3 Beginning of the Match – Best of 3 or Best of 5

Another natural point for our analysis is the very beginning of the entire match. During Grandslam tournaments, women play matches decided by the best of three sets, while men play matches decided by the best of five sets. Both situations are illustrated in Figures 4 and 5 respectively.

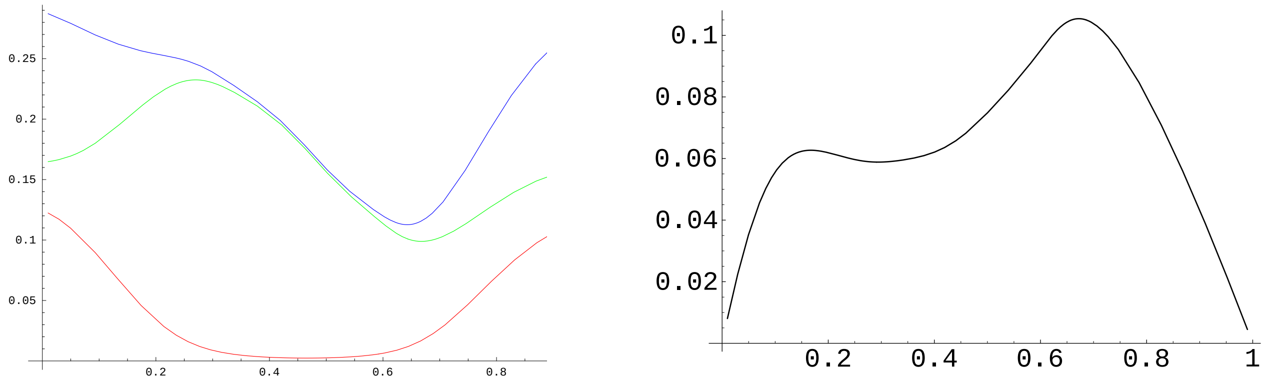


Figure 4: Beginning of the match of best of 3 sets (women's game).

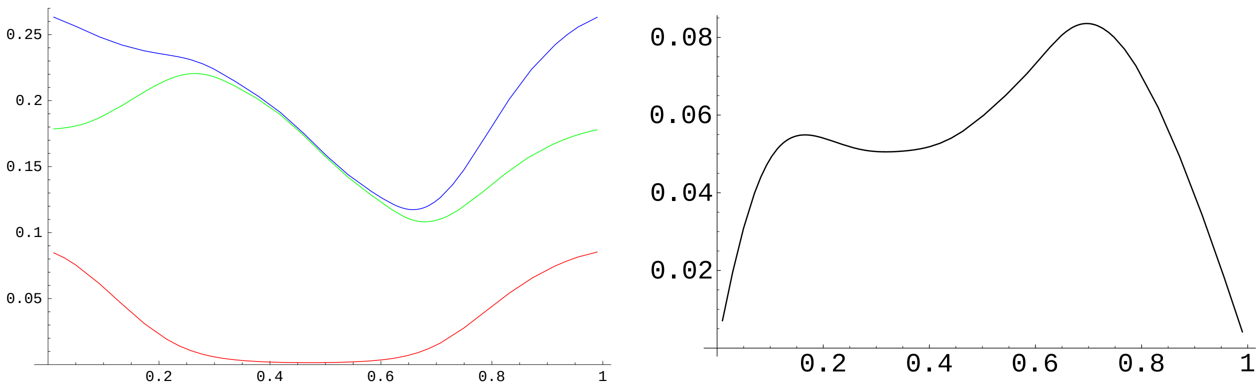


Figure 5: Beginning of the match of best of 5 sets (men's game).

Obviously the difference between playing match with tiebreak in the last set in contrast to playing match with no tiebreak in the last set is much smaller in the beginning of the entire match. The most extreme difference in the best of 3 match is at most 1% if the less consistent player's winning probability on serve falls within interval between 30% and 65%. It is typical situation in women's tennis that the less consistent player probability fits into that situation. The winning chances of the each player would be thus only negligently affected by adopting tiebreak in the decisive set.

In men's tennis, serving consistencies can reach higher values and the match can become a little more sensitive to the rule of playing tiebreak in the last set. However, this effect can change the winning chances by at least one percentage point only when the consistency of the least consistent player exceeds 66%, which is at the higher end of professional play. For instance, if the least consistent player has 67% winning chances on serve and the more consistent player has 75%, the chances of winning the match by the worse player are around 12.5% for the match with tiebreak in the last set and 11.5% for the match without the tiebreak. In more typical regions, the difference in playing tiebreak is much more negligent.

## 4 Conclusion

Playing tiebreak in the last set is always beneficial for the less consistent player. The closer the match gets to the 6:6 score in the last set, the more pronounced is the difference between the scenarios of playing or not playing the tiebeak. At the beginning of the match, this difference for players with typical serving consistency is typically far smaller than 1%. However, for highly consistent players, this difference can get slightly greater as illustrated by the above analysis.

## References

[1] Lawler, G., *Introduction to Stochastic Processes,* Chapman & Hall, 1995.