Valuing Simple Multiple-Exercise Real Options in Infrastructure Projects

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Abstract: The revenue risk is considerable in infrastructure project financing arrangements such as build–operate–transfer (BOT). A potential mitigation strategy for the revenue risk is a governmental revenue guarantee, where the government secures a minimum amount of revenue for a project. Such a guarantee is: (1) only redeemable at distinct points in time; and (2) more economical if the government limits the guarantee's availability to the early portions of a BOT's concession period. Hence, a guarantee characterized by this type of structure takes the form of either a Bermudan or a simple multiple-exercise real option, depending upon the number of exercise opportunities afforded. The multi-least-squares Monte Carlo technique is presented and illustrated as a promising approach to determine the fair value of this variety of real option. This method is far more flexible than prevailing approaches, so it represents an important step toward improving risk mitigation and facilitating contractual and financial negotiations in BOT projects.

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Introduction

The momentum worldwide toward greater utilization of private capital for public purpose, which began in the latter stages of the 20th century, shows no sign of abating. In the United States, 20 states have enabling legislation that permits some form of public–private initiatives for transportation projects (Reinhardt 2004). Internationally, the Private Finance Initiative (PFI) in the United Kingdom is well known while the use of private capital for infrastructure projects within emerging economies has become a global trend where financially challenged public administrations look toward the private sector to develop basic infrastructure (Esty 2003). Without a marked change in the allocation of budgetary resources by governments at all levels, the share of infrastructure that is privately financed will certainly increase.

Effectively, the public and private sectors are fashioning new partnership arrangements where the efforts of the two sectors are combined to achieve a common objective, the successful realization of an infrastructure project. Certainly, these two parties are motivated by different objectives; the public sector is interested in the political and socio-economic benefits that a successful project can provide while the private sector is primarily concerned with the financial profitability of a project.

One of the more effective methods to exploit this public–private partnership is the build–operate–transfer (BOT) project delivery method. Private participation in a BOT delivery strategy, however, is conditioned upon the mitigation of the risks that may adversely impact a project's profitability. A relevant BOT project risk that may seriously undermine a project's profitability is the revenue risk, that is, "the risk that the project may not earn sufficient revenue to service its operating costs and debt and leave an adequate return for investors" (Yescombe 2002). If the private participants do not feel comfortable with the level of revenue risk, they will typically withdraw from a BOT project.

The public sector, i.e., the government, could provide incentives or subsidies to entice private investment. These incentives might take the form of "guarantees" where the government secures a minimum amount of revenue in order to improve the creditworthiness of a BOT arrangement. In effect, the government has granted the sponsor a contract to cover the revenue shortfall over a specific operating period. If the concept of a revenue guarantee is generalized, then a revenue guarantee is a contract in which one party, the guarantor, promises to pay the other party, the third party guaranteed (TPG) the revenue shortfall $(K-X)$ relative to a period of time Δt, that is the difference between the minimum guaranteed net revenue, K, and the net revenue accumulated in Δt, X. The contractual cumulative period Δt=project financial auditing interval, which typically occurs on a quarterly, semiannual or annual basis. Note that this type of scheme is akin to a financial put option.

The nature of a revenue guarantee is fully determined once the following two elements are established:

1. Number of exercise rights, M, representing the number of times the TPG is entitled to exercise or "redeem" the guarantee; and
2. Number of exercise dates, N, representing the dates where the M exercise rights can be executed. At each date, it is possible to execute only one exercise right. The exercise

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dates are set at Δt period apart from each other.

Theoretically, the structure of a revenue guarantee may take various forms. For example, the guarantor could grant the TPG: (1) the same number of exercise rights as exercise dates, $M = N$; and (2) fewer exercise rights than exercise dates, $M < N$; or (3) a single exercise right, $M = 1$.

Clearly, this "real" option has value. If the value of such an option is substantial and no effort is made to quantify it, then the government may unknowingly provide the sponsor a tremendous subsidy. Alternatively, the sponsor may unwittingly disregard or attach a conservative value to the option in view of its vagueness. In the absence of an objective measure to reconcile the expectations between the two parties, abandonment of a needed project is a likely possibility, thus leading to a lose–lose situation.

Accordingly, the intent of this paper is to present a method for quantifying the value of a revenue guarantee in a BOT project. The valuation approach extends existing computational finance methods used to price discrete-exercise financial options, specifically the least-squares Monte Carlo method, and the approach is illustrated in a hypothetical case study. The valuation method presented is far more flexible than prevailing methods, and it can provide governments, sponsors, and lenders the ability to determine the worth of an important revenue risk mitigation strategy, a possibility that heretofore was only notional.

Background

Option Theory

The basic structure of a revenue guarantee suggests that real options analysis techniques are appropriate for its valuation. Generally, an option may be defined as the opportunity to take a beneficial action, within a bounded time frame, when a favorable condition occurs. Accordingly, option theory studies how to model and price this "opportunity" which is typically either a contractual right (e.g., financial options, flexible commodity contracts) or system flexibility (e.g., expansion or delay options).

Option theory embraces two principal research fields: financial option theory and the more recent real option theory. The former refers to option theory applied to assets traded in the financial markets, while the latter concerns option theory applied to nonfinancial assets (or real assets). The foundation for financial option theory was established in 1900 by the French mathematician Louis Bachelier, and this field matured in the 1970s thanks to the seminal Nobel Prize research by Merton, Black, and Scholes (Merton 1973; Black and Scholes 1973). Alternatively, real option theory is a relatively recent development and Myers (1977), who first coined the term "real option," is credited for its initiation. Over the years, other researchers have significantly contributed to expand real option theory (Dixit and Pindyck 1994; Trigeorgis 1996; Amram and Kulatilaka 1999; Copeland and Antikarov 2001). Wang and de Neufville (2005) clarified the "nature" of real options by categorizing them into real options "on" and "in" projects. Real options "on" projects are mostly concerned with the valuation of investment opportunities, while real options "in" projects are mostly concerned with the design of flexibility (Wang and de Neufville 2005).

Several authors have investigated the use of real option analysis in infrastructure problems: Ford et al. (2002) in strategic planning of a toll road project; Ho and Liu (2002) in evaluating A/E/C technology investments; Garvin and Cheah (2004) in infrastructure investment decisions; Zhao et al. (2004) in highway development; Zhao and Tseng (2003), along with de Neufville et al. (2006), in designing multistory parking garages.

Table 1. European, Bermudan, and Australian Options

European option	An option that can be exercised one time, only at the end of its life
Bermudan option	An option that can be exercised one time, on specified dates during its life.
Australian option (simple multiple-exercise option)	An option than can be exercised M times, on specified N $(N \geq M)$ dates during its life

Discrete-Exercise Options

A revenue guarantee is a particular type of "real" option, a discrete-exercise option. Discrete exercise options are ones that can be exercised at discrete points over a predetermined time period. Discrete-exercise options generally take one of three forms: European, Bermudan, and simple multiple-exercise options. Table 1 differentiates the characteristics of each. Both the European and Bermudan option classes are related to the simple multiple-exercise option class (Jaillet et al. 2004). Hereafter, simple multiple-exercise options are also referred to as "Australian" options.

Valuing a European option is rather straightforward, even by Monte Carlo simulation; however, pricing of Bermudan and Australian options through simulation is more challenging. A decade ago, most academics assumed that it was not possible to employ Monte Carlo simulation to price Bermudan options. In the last 10 years, several researchers have successfully combined different dynamic programming techniques with Monte Carlo simulation to value financial Bermudan options: Broadie and Glasserman (1997), Andersen (2000), and Longstaff and Schwartz (2001). The Longstaff–Schwartz approach, also referred to as the least-squares Monte Carlo (LSM) method, is the most popular approach among them, mainly for its mark of being an intuitive and flexible tool (Gamba 2003).

Pricing Bermudan options is complicated because of the right of the holder to exercise the option at multiple, yet finite, points in time before maturity. At each exercise point, the holder optimally compares the cash flow due to an immediate exercise, the *immediate exercise value*, with the cash flow generated if the option is exercised in the future, the *continuation value*. Thus, the optimal exercise decision relies upon estimating the continuation value. Longstaff and Schwartz (2001) proposed estimating the continuation value by a least-squares regression together with the cross-sectional information provided by a Monte Carlo simulation. The continuation value function is then approximated by fitting the future cash flows using, for example, simple polynomial basis functions. Comparing these estimates with the cash flows of immediate exercise yields the optimal stopping rule. By expanding the scope of the investigation, the LSM approach can also be extended to multiple-exercise (Australian) options. Literature on the LSM method to evaluate Bermudan options is quite extensive, including the seminal Longstaff and Schwartz article (2001), Clement and Protter (2002), Moreno and Navas (2003), Gamba (2003), and Haugh (2003). On the other hand, literature on the LSM method applied to multiple-exercise options appears limited to Meinshausen and Hambly (2004).

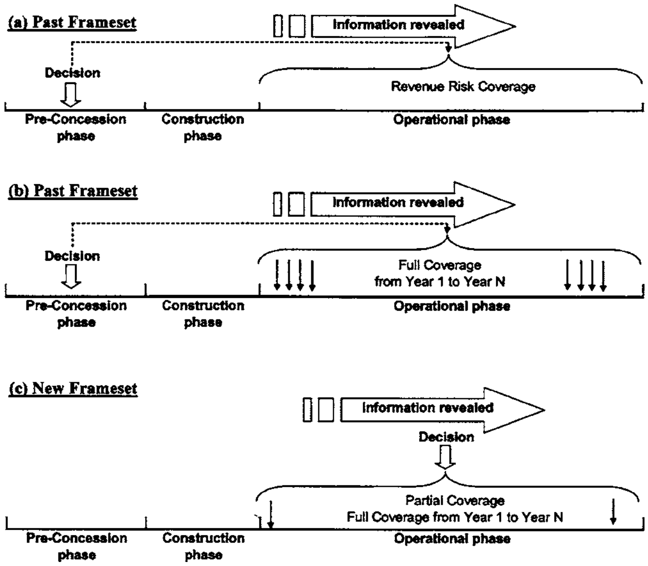

Fig. 1. Past and new framesets for revenue guarantees: (a) partial coverage, exercise dates defined before operational period; (b) full coverage, exercise dates defined before operational period; and (c) partial or full coverage, exercise dates determined during operational period

Valuing Revenue Guarantees

Current Approaches

While the concept of a revenue guarantee as a revenue risk mitigation strategy for BOT projects is appealing, the current techniques for establishing the fair value of the various configurations that revenue guarantees might take are quite limited. Dailami et al. (1999) developed a valuation method for a revenue guarantee in BOT project settings using Monte Carlo simulation, but the method presumes that the TPG has the right to redeem the guarantee at the end of each timeframe of a concession period, i.e., N=length of concession period (years), $\Delta t=1$ year, and $N=M$. This structure provides the TPG with full revenue risk coverage, and Dailami et al.'s approach values this full coverage guarantee as a stream of European put options. In other words, the fair price of such a contract is the value of N European put options with maturity set at the end of each operational year with a payoff of

$$\Pi(X) = \max(K - X, 0) \tag{1}$$

Alternatively, Irwin (2003) introduced a simpler technique for valuing a revenue guarantee with the following general structure: (1) $N=M=1$; and (2) $\Delta t=y$, where y=period of time fixed by the guarantor. In this case, the guarantor is offering the TPG a single opportunity to redeem the guarantee at a predetermined date. Irwin determines the value of this guarantee by treating it as a single European put option and employing the familiar Black–Scholes equation to price the option.

The major shortcoming of Irwin's valuation approach is that the TPG must specify beforehand the exact years in which he wants to exercise the revenue guarantee. Therefore, relevant information about the revenue guarantee shortfalls that will be eventually revealed during the operational phase cannot be used by the TPG [Fig. 1(a)]. On the other hand, this lack of decision flexibility may be overcome by arranging for a full risk coverage revenue guarantee that protects the TPG against the revenue

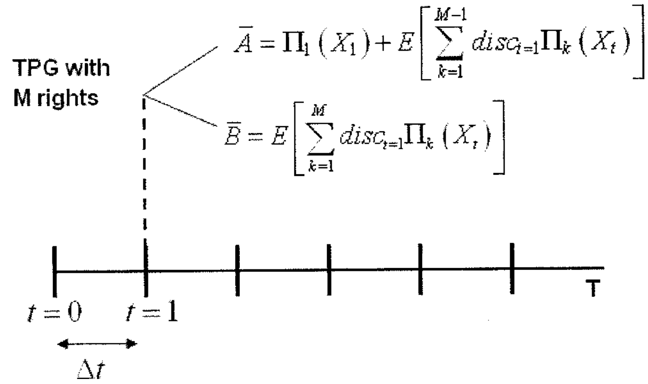

Fig. 2. TPG's decision making process

shortfalls over all operational years [Fig. 1(b)]. In this case, Dailami et al.'s valuation technique may be used; however, this type of arrangement is likely to result in a costly long-term commitment for the guarantor.

A new "dynamic" valuation frameset is needed, which is unlike the above mentioned revenue guarantee arrangements that show "static" decision features. A dynamic approach would be able to value a revenue guarantee where the TPG decides on the "spot" i.e., during the operational phase, whether or not to redeem the revenue guarantee. Thus, the TPG could take full advantage of the relevant information that will be revealed over the operational phase to make the best exercise decision [Fig. 1(c)].

Proposed Framework

The value of the "dynamic" revenue guarantee contract with M claims [Fig. 1(c)] is the expected amount of dollars that the guarantor will have to pay the TPG as a result of the TPG's exercise policy, i.e., execution of the M claims. Different exercise policies return different values of the contract; however, only one exercise policy, the "optimal" exercise policy, will generate the maximum profit for the TPG. Accordingly, the fair value of the contract can be defined as the expected amount of dollars to be paid by the guarantor if the TPG executes an "optimal" exercise policy.

The basic idea is to model the decision-making behavior of the TPG to determine if and when the TPG will choose to redeem or exercise revenue guarantee rights. This provides an estimate of the fair value of the revenue guarantee option since the approach has determined the potential "cost" of a revenue guarantee to the guarantors. The valuation framework presumes that the TPG is a profit maximizer, so given two dollar amounts A and B where $A > B$, this party will always prefer A. In addition, the TPG cannot foresee the future; thus, the TPG's forecast of future revenues is an expectation as opposed to a certainty.

Defining the stochastic evolution of the cumulative net revenue X over time is the most crucial element of the entire modeling process. This framework represents the net revenue X as a discrete-time stochastic process that spans the operational period. In this type of setting, the modeler may use either a discrete-time one-factor model (Irwin 2003) or a discrete-time multifactor model (Dailami et al. 1999).

Once the analyst defines the model for the underlying variable, X, the evolution of the net revenue shortfall over time can be represented by the payoff function Eq. (1), which is a discrete-time stochastic process depending on the minimum net revenue, K, and the cumulative net revenue, X.

Once the salient features of the revenue guarantee contract are determined, the TPG has acquired a revenue guarantee contract with M exercise rights at time $t=0$. The decision making process followed by the TPG is illustrated in Fig. 2. At the end of the first unit time period ($t=1$), TPG must decide:

1. Whether to exercise one of the M exercise rights, the return being the sum of the payoff due to one exercise and the expected value of the M-1 future exercise payoffs, discounted to $t=1$ (i.e., $\mathrm{disc}_{t=1}$)

$$\bar{A} = \Pi_1(X_1) + E\left[\sum_{k=1}^{M-1} \mathrm{disc}_{t=1}\Pi_k(X_t)\right] \tag{2}$$

2. Or not to exercise, the return being the expected value of M future exercise payoffs, discounted to $t=1$

$$\bar{B} = E\left[\sum_{k=1}^{M} \mathrm{disc}_{t=1}\Pi_k(X_t)\right] \tag{3}$$

where the subscript k refers to the kth claim of the M-1 remaining claims (i.e., $k=1,2,\ldots,M-1$); while the subscript t refers to the timestep in which it is still possible to exercise one of the remaining M-1 claims (i.e., $t=2,3,\ldots,N$).

If TPG expects that $\bar{A}>\bar{B}$ he will choose $\bar{A}$, i.e., he will exercise the right and will go to the next time step with M-1 rights remaining. Otherwise, if TPG supposes that $\bar{A}<\bar{B}$, he will choose $\bar{B}$, i.e., he will not exercise a right and will go to the next time step with M rights remaining. The TPG's decision-making process continues until either the expiration date of the revenue guarantee or the timestep in which the Mth right is executed.

The decision process presented in Fig. 2 is a multistage decision process with a return associated to each decision. The objective of the guarantor in analyzing such a process is to determine the TPG's optimal decision policy, that is the collection of distinct points in time (the optimal stopping time set) that results in the best total return for TPG

$$\{\tau_k^M\} = (\tau_1^M, \tau_2^M, \ldots, \tau_M^M) \tag{4}$$

Once the optimal stopping time set is determined, the fair value of the revenue guarantee, $\hat{\lambda}$, is given by the expectation of the sum of the payoffs relative to the optimal stopping time set, discounted to $t=0$

$$\hat{\lambda} = E\left[\sum_{k=1}^{M} \mathrm{disc}_{t=0}\Pi_k\left(X_{\tau_k^M}\right)\right] \tag{5}$$

When the cumulative net revenue X is modeled using either a geometric Brownian motion, a Brownian motion, or their derivative processes (e.g., Ornstein–Uhlenbeck process), the true value $\hat{\lambda}$ can be computed by extending the binomial or trinomial tree method to a multiple-exercise option framework (Jaillet et al. 2004).

However, a realistic representation of the net revenue evolution over time may require a more complex stochastic model such as a multifactor model. In this case, a binomial or trinomial tree method to value the multiple-exercise option is computationally impractical. An alternative and effective approach is to combine Monte Carlo simulation with dynamic programming techniques and use it to compute the revenue guarantee value. The value of a revenue guarantee that permits the redemption of M claims can be framed as an Australian option with M exercise rights and payoff function Eq. (1). The valuation method to assess an Australian option was obtained by extending the Longstaff and Schwartz (2001) approach to a multiple-exercise option framework.

Valuation Framework for an Australian Option

An Australian option is a discrete-time American-type option which allows the redemption of M claims, $M\geq 1$, at different stopping times $0\leq t_1^M, t_2^M, \ldots\ldots, t_M^M \leq T$ chosen by the holder of the option.

Hence, an Australian option is exercisable M times within the interval $[0,T]$ and the possible N exercise dates are $\{t_1=\Delta t, t_2=2\Delta t, \ldots\ldots, t_N=T=N\Delta t\}$ with time step $\Delta t=T/N$. The dynamics of the state variables are simulated by generating n paths $\{X_{t_1}^i, X_{t_2}^i, \ldots, X_{t_j}^i, \ldots, X_{t_N}^i\}_{i=1,2,\ldots,n}$. The resulting $X_{t_j}^i$=value of the process at time $t_j=j\Delta t$ along the ith simulated path. Hereafter, the discount rate r is considered constant for clarity of exposition.

To assess the value of the option for each simulated path a backward dynamic programming approach is employed. Based on the Bellman's "principle of optimality" (Powell 2005), the value of the Australian option with M exercise rights at time t_j is given by the Bellman equation

$$\underset{(M)}{V}(t_j, X_{t_j}^i) = \max\{\bar{A}, \bar{B}\} \tag{6}$$

where

$$\bar{A} = \Pi(t_j, X_j^i) + \underset{(M-1)}{\bar{C}}(t_j) = \Pi(t_j, X_j^i) + E[e^{-r\Delta t}\underset{(M-1)}{V}(t_{j+1}, X_{j+1})] \tag{7}$$

=return if the option holder decides to exercise one of the M exercise rights at time t_j, that is the sum of $\Pi(t_j, X_j^i)$, the payoff due to one exercise, and $\underset{(M-1)}{\bar{C}}(t_j)$, the continuation value with M-1 exercise rights. $\underset{(M-1)}{\bar{C}}(t_j)$=expected value of the Australian option with M-1 exercise rights discounted to t_j (e.g., using the discount factor $e^{-r\Delta t}$), and

$$\bar{B} = \underset{(M)}{\bar{C}}(t_j) = E[e^{-r\Delta t}\underset{(M)}{V}(t_{j+1}, X_{j+1})] \tag{8}$$

=return if the option holder decides not to exercise any one of the M exercise rights at time t_j. The continuation value with M exercise rights, $\underset{(M)}{\bar{C}}(t_j)$=expected value of the Australian option with M exercise rights discounted to t_j.

The difficulty in calculating Eq. (6) lies on the assessment of the continuation value in Eqs. (7) and (8). A way to overcome this difficulty is to consider the continuation value as an expectation conditioned to the information known at time t_j

$$\underset{(K)}{C}(t_j) = E[Y_j^{(K)} | X_j^i] \tag{9}$$

where $\underset{(K)}{C}(t_j)$=continuation value relative to an Australian option with K exercise rights, and

$$Y_j^{(K)} = e^{-r\Delta t}\underset{(K)}{V}(t_{j+1}, X_{j+1}) \tag{10}$$

=value of the option with K exercise rights discounted to t_j; and X_j=known underlying variable at time $t=t_j$. The continuation value in Eq. (9) can be approximated by adopting the least-squares regression approach of Longstaff and Schwartz (2001). Then, Eq. (9) can be represented as a linear function of the elements of an orthonormal countable basis

$$\underset{(K)}{C}(t_j,X_j) = E[Y_j^{(K)}|X_j] = \sum_{k=1}^{\infty} a_k(t_j)p_k(t_j,X_j) \tag{11}$$

where p_k=kth element of the basis; and a_k=associated constant coefficient. If Eq. (11) is represented using a finite number of elements of the orthonormal basis ($Z<\infty$), then an approximation of the continuation value Eq. (9) is given by

$$\underset{(K)}{C}(t_j,X_j) = E[Y_j^{(K)}|X_j^i] \approx \sum_{k=1}^{Z} a_k(t_j)p_k(t_j,X_j) \tag{12}$$

The set of coefficients $\{a_k(t_j)\}$ can be estimated by least-squares regressing the simulated values of Eq. (10) onto the basis

$$[\hat{a}_k(t_j)]_{k=1}^{Z} = \arg\min\left\|\sum_{k=1}^{Z} a_k(t_j)p_k(t_j,X_j^i) - e^{-r\Delta t}\underset{(K)}{V}(t_{j+1},X_{j+1}^i)\right\| \tag{13}$$

So, the estimated continuation value is

$$\underset{(K)}{\hat{C}}(t_j,X_j) = \sum_{k=1}^{Z} \hat{a}_k(t_j)p_k(t_j,X_j) \tag{14}$$

The Bellman Eq. (6) at time step t_j for the ith simulated path can be rewritten as

$$\underset{(M)}{V}{}^i(t_j,X_{t_j}^i) = \max\{\bar{A}^*,\bar{B}^*\} \tag{15}$$

where

$$\bar{A}^* = \Pi(t_j,X_j^i) + \underset{(M-1)}{\hat{C}}(t_j,X_j^i) \tag{16}$$

=return if the option holder decides to exercise one of the M exercise rights at time t_j; and

$$\bar{B}^* = \underset{(M)}{\hat{C}}(t_j,X_j^i) \tag{17}$$

=return if the option holder decides not to exercise any one of the M exercise rights at time t_j.

Therefore, the decision rule at time step t_j along the ith path is given by

$$IF\ \Pi(t_j,X_j^i) + \underset{(M-1)}{\hat{C}}(t_j,X_j^i) > \underset{(M)}{\hat{C}}(t_j,X_j^i)$$
$$THEN\ t_j \text{ is an optimal stopping time} \tag{18}$$

When immediate exercise is performed at time t_j, the optimal stopping time set for $t\geq t_j$ must be rearranged

$$\{\tau_k^M\}_{t\geq t_j} = t_j \cap \{\tau_k^{M-1}\}_{t\geq t_{j+1}} \tag{19}$$

that is the optimal stopping set for $t\geq t_j$ relative to the Australian option with M exercise rights given by the optimal stopping time t_j and the optimal stopping time set for $t\geq t_{j+1}$ relative to the Australian option with M-1 exercise rights.

It can be inferred from Eq. (19) that the optimal stopping time set $\{\tau_k^M\}_{t\geq t_j}$ is completely resolved when information of the optimal stopping time set relative to an Australian option with M-1 exercise rights, $\{\tau_k^{M-1}\}_{t\geq t_{j+1}}$, is known. Moreover, it can be shown that the optimal stopping time set relative to an Australian option with y exercise rights, $\{\tau_k^y\}$, depends on information of the optimal stopping time set of an Australian option with one exercise less, $\{\tau_k^{y-1}\}$, as shown in Fig. 3.

Eventually, in order to solve the optimal stopping time set $\{\tau_k^M\}$, the analyst must run additional M-1 optimization procedures applied to the same Australian option with exercise rights ranging from 1 to M-1. Proceeding backward with the dynamic programming approach, we can calculate the optimal stopping time set $\{\tau_1^M,\tau_2^M,\ldots,\tau_M^M\}$ for the ith simulated path. Once the optimal stopping time set is known, it is possible to calculate the option value for the ith simulated path

$$\underset{(M)}{V}{}^i(0,X^i) = \sum_{k=1}^{M} e^{-r\tau_k^M}\Pi(\tau_k^M,X_{\tau_k^M}^i) \tag{20}$$

If the same procedure is applied to all n simulated paths, then the estimate of the Australian option value may be computed as

$$\hat{\theta} = \underset{(M)}{\hat{V}}(0,X) = \frac{1}{n}\sum_{i=1}^{n}\underset{(M)}{V}{}^i(0,X^i) = \frac{1}{n}\sum_{i=1}^{n}\sum_{k=1}^{M} e^{-r\tau_k^M}\Pi(\tau_k^M,X_{\tau_k^M}^i) \tag{21}$$

The option estimate Eq. (21) is a low biased estimate of Eq. (5), i.e., $\hat{\theta}\leq\hat{\lambda}$, because we have approximated the continuation value in Eq. (15). Ideally, the value of the Australian option is determined by a stopping rule that maximizes the value of the option. The LSM method, however, computes an approximated estimate of the continuation value both because we consider only a finite number of elements of the orthonormal basis ($Z<\infty$) and because we substitute the set of coefficients $\{a_k(t_j)\}$ with their approximation $\{\hat{a}_k(t_j)\}$. Eventually, using the approximated estimate of the continuation value, $\underset{(K)}{\hat{C}}$, instead of its true value, $\underset{(K)}{C}$, leads to a stopping rule that is less than or equal to the optimal one, i.e., a suboptimal stopping rule. Accordingly, the computed estimate of the option value will be biased low.

Hypothetical Case Example

The approach to value an Australian option is now applied to value a limited revenue guarantee within a BOT toll road project with a 30-year concession period. The project's total capital expense is \$160 million, and \$120 million is provided by senior debt and \$40 million is provided by equity. A simplified cash flow model (Esty 1999) to determine the project's annual net revenue available to the sponsors, i.e., annual equity cash flow (ECF), is given by

$$\text{ECF}_i = \text{gross revenue}_i\text{-total cost}_i\text{-tax}_i\text{-debt service}_i \tag{22}$$

The cost of equity and the borrowing interest rate considered in this example are 15 and 10%, respectively. The project's base case financial projections as well as the NPV computations are presented in Table 2. The base case yields a NPV of roughly \$4.3 million and an IRR of 19.7%.

A Monte Carlo simulation risk analysis is then performed where traffic volume is the only risk variable considered. The traffic volume is modeled as a random variable with a dynamic variance. This representation of the traffic volume uses a variance model, which has a deterministic component, the expected value vector of the traffic volume, and a random component, the uncertainty of the process. See Chiara (2006) for further details. In this case, the column of the average daily traffic values in Table 1 is assumed to be the vector of the forecasted traffic volume, $\bar{\mathbf{W}}=[\bar{W}_1,\bar{W}_2,\ldots,\bar{W}_T]$. The uncertainty of the process can be de-

Table 2. Case Example Valuation Model

	Financial projections							Valuation	
Period	Avgerage daily traffic	Toll per vehicle ($)	Gross revenue ($)	Total cost (operation, maintenance, etc.) ($)	Tax ($)	Debt service ($)	Captial expenditure ($)	ECF ($)	Discounted ECF ($)
1							(100,000,000)		
2							(60,000,000)		
3	30,000	1.50	16,425,000	(7,000,000)	0	(4,000,000)		5,425,000	3,567,026
4	32,100	150	17,574,750	(7,210,000)	(1,227,500)	(4,000,000)		5,137,250	2,937,239
5	34,347	150	18,804,983	(7,426,300)	(1,509,425)	(4,000,000)		5,869,258	2,918,058
6	36,751	150	20,121,331	(7,649,089)	(1,813,605)	(4,000,000)		6,658,638	2,878,713
7	39,324	150	21,529,824	(7,878,562)	(2,141,673)	(4,000,000)		7,509,590	2,823,133
8	42,077	200	30,715,883	(8,114,919)	(2,495,379)	(14,274,472)		5,831,113	1,906,201
9	43,339	200	31,637,359	(8,358,366)	(897,948)	(13,874,472)		8,506,574	2,418,099
10	44,639	200	32,586,480	(8,609,117)	(1,221,356)	(13,474,472)		9,281,535	2,294,253
11	45,978	200	33,564,075	(8,867,391)	(1,550,867)	(13,074,472)		10,071,345	2,164,767
12	47,358	200	34,570,997	(9,133,412)	(1,886,664)	(12,647,472)		10,876,449	2,032,886
13	48,778	200	40,059,143	(9,407,415)	(2,228,934)	(20,274,472)		8,148,322	1,324,330
14	50,242	225	41,250,917	(9,689,637)	(3,913,177)	(19,474,472)		8,183,631	1,156,582
	—	—	—	—	—	—		—	—
	—	—	—	—	—	—		—	—
29	78,275	250		15,096,139	14,293,743	9,156,814		32,879,043	571,060
30	80,623	250		15,549,023	15,034,177	8,329,366		34,655,943	523,411
31	83,042	250		16,015,494	15,789,378	7,501,919		36,468,774	478,948
32	85,533	250		16,495,959	27,400,951	6,674,472		27,477,450	313,795
									44,293,530

Note: NPV(ECF)=PV(ECF)-PV(WQUITY)=44,293,530-40,000,000=$4,293,530 IRR=19.7%. (a) Cost of equity Ke=15%. (b) Borrowing Interest rb =10%. (c) Initial average daily traffic: 30,000 vehicle; traffic growth: 7% b/w year 3 and 9 and 3% b/w 10 and 32.

fined by three parameters: an independently distributed random sequence with a mean of zero and a unit variance $\{\varepsilon_t\}$, a variance function $H(t)$, and the variance of the process at $t=1$, σ^2. In this case, the random sequence is defined as $\{\varepsilon_t\}\sim\text{beta}(a=3,b=3)$, the variance function is defined as $H(t)=\sigma^2 t$, and the variance σ^2 is estimated by considering a lower and an upper bound of the traffic forecast relative to year one, low=25,000 vehicles and high=35,000 vehicles, respectively. The risk analysis generates a NPV distribution with an expected NPV of $4.3 million and the probability that NPV is negative of 36.3%.

With such a high likelihood that the project has a negative NPV, the government could choose to offer the sponsor a revenue guarantee with multiple-exercise opportunities. This multiple-exercise governmental guarantee is modeled as an Australian option with payoff

$$\Pi(t,\text{ECF}) = \max(K - \text{ECF}, 0) \tag{23}$$

where K=amount of revenue that the government pledges to guarantee, so if the annual equity cash flow is less than K, then the government will pay the difference.

A minimum equity cash flow equal to K generates a sponsor's rate of return of 15%, i.e., as much as the cost of equity.

The fair price of a multiple-exercise governmental guarantee covering the revenue risk for x years (which can be nonconsecutive years) is given by the value of an Australian option with payoff Eq. (23) and $M=x$ exercise rights. With $K=\$6.5$ million, a real option analysis is performed varying both the number of exercise rights, M, and number of simulations. A discount factor with a discretely compounded discount rate, $1/(1+r)^{\Delta t}$, was used in the option computations. The assumed discount rate, r, which depends on the presumed credit risk of the government, was chosen as $r=6\%$. The least-square regression was performed using $\{1, X, X^2\}$ as basis functions. The results of the real option analysis for different numbers of simulated paths are shown in Table 3.

With respect to the values relative to 100,000 simulated paths, the revenue risk may be quantified as the expected value of the ECF shortfalls over the entire operational period (i.e., project revenue risk=$8.169 million). As much as 99% of the revenue risk (i.e., $ 8.169 million × 0.99 = $ 8.08 million) can be mitigated by a revenue guarantee with 15 exercise rights. Furthermore, a revenue guarantee with eight exercise rights can mitigate as much as 80% of the project revenue risk (i.e., $8.169 million × 0.80 = $ 6.53 million), and a revenue guarantee with four exercise rights can mitigate as much as 54% of the project revenue risk (i.e., $8.169 million × 0.54 = $ 4.41 million). These results strongly suggest that, before providing a full coverage guarantee, the Government and the sponsor should perform an Australian option analysis to determine alternative guarantee structures that may better fit their revenue risk mitigation strategy.

Conclusion

A revenue guarantee in a BOT project takes the form of a discrete-exercise real option. It may be valued by treating the guarantee as one of three classes of discrete-exercise options: European, Bermudan, and simple multiple-exercise (Australian) options. Current valuation approaches represent the guarantee as a European option where *the quantity and the time* of exercise opportunities must be specified beforehand. Such "static" contracts do not permit the TPG to make use of information as it is revealed during the operational period. Moreover, if these "static con-

Table 3. Multiple-Exercise Guarantee Analysis

Option rights M	n. simulations 1,000	5,000	10,000	50,000	100,000
1	1.322[a]	1.302	1.294	1.318	1.318
2	2.526	2.525	2.541	2.545	2.545
3	3.543	3.583	3.561	3.588	3.581
4	**4.389**	**4.462**	**4.435**	**4.466**	**4.459**
5	5.023	5.128	5.08	5.122	5.111
6	5.532	5.666	5.616	5.667	5.654
7	5.967	6.145	6.087	6.144	6.13
8	6.338	6.552	6.486	6.551	6.536
9	6.702	6.908	6.829	6.903	6.887
10	6.975	7.223	7.129	7.21	7.195
11	7.208	7.748	7.412	7.489	7.47
12	7.440	7.938	7.641	7.711	7.697
13	7.610	8.083	7.826	7.890	7.874
14	7.752	8.187	7.966	8.027	8.011
15	**7.853**	**8.208**	**8.066**	**8.120**	**8.106**
16	7.879	8.222	8.086	8.139	8.125
17	7.898	8.233	8.100	8.152	8.138
18	7.912	8.240	8.111	8.161	8.147
19	7.921	8.245	8.118	8.168	8.153
20	7.931	8.249	8.124	8.173	8.158
21	7.938	8.251	8.129	8.177	8.162
22	7.942	8.252	8.132	8.18	8.165
23	7.945	8.253	8.135	8.181	8.166
24	7.946	8.254	8.137	8.184	8.167
25	7.947	8.254	8.138	8.183	8.167
26	7.948	8.254	8.138	8.184	8.168
27	7.948	8.254	8.138	8.184	8.168
28	7.948	8.254	8.138	8.184	8.169
29	7.948	8.254	8.138	8.184	8.169
30	**7.948**	**8.254**	**8.138**	**8.184**	**8.169**

[a]$million.

tracts" are arranged to fully cover the revenue risk, they become a costly long-term commitment. The multi-least-squares Monte Carlo method was developed and presented to support the structuring of a "dynamic contract" that allows BOT project participants to be extremely flexible in dealing with the revenue risk during the project operational phase.

This novel valuation framework has been developed by extending current option theory. Specifically, the work expands the least-squares Monte Carlo technique, the LSM method, to value Australian real options. An Australian option has embedded M claims that can be redeemed at M stopping times $\{\tau_1^M, \tau_2^M, \ldots, \tau_M^M\}$, which are chosen by the holder of the option. Presumably, the option holder will choose the stopping times that maximize the value of the option. An option holder at any time step t must decide whether to redeem one of the claims immediately or to wait and redeem all of them in the future. This multistage decision process has a return associated with each decision, and the objective in analyzing such a process is to determine an optimal policy, one that results in the best total return. The value associated with this best total return is then the option value.

The techniques were illustrated using a hypothetical case study. The case example illustrated the application of Australian options to mitigate the revenue risk of BOT toll road project with a concession period of 30 years. As shown, 99% of the revenue

$$\{\tau_k^M\} = f_{M-1}\left(\{\tau_k^{M-1}\}\right)$$
$$\{\tau_k^{M-1}\} = f_{M-2}\left(\{\tau_k^{M-2}\}\right)$$
$$\cdots\cdots\cdots\cdots$$
$$\cdots\cdots\cdots\cdots$$
$$\{\tau_k^3\} = f_2\left(\{\tau_k^2\}\right)$$
$$\{\tau_k^2\} = f_1\left(\{\tau_k^1\}\right)$$

Fig. 3. Recursive information chain of optimal stopping time sets

risk is mitigated with a governmental guarantee covering 15 years, or half of the concession period. Moreover, a governmental guarantee covering only 4 years can mitigate the revenue risk by 54%. These results have important implications for the BOT market. Foremost, credible methods to value revenue guarantees can improve the assessment and allocation of risks in BOT projects. Such methods will permit governments, lenders, and sponsors to determine the fair value of this risk mitigation strategy, which can preclude conferring substantial subsidies or undervaluing investment opportunities. Additionally, the valuation techniques illustrated could lead to the development of a secondary market for revenue guarantees where single or multiple party guarantors provide revenue "insurance" in exchange for a premium. Both possibilities are important developments for the expanding BOT marketplace.

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