



PARALLEL COORDINATES: *Visual* MULTIDIMENSIONAL GEOMETRY AND ITS APPLICATIONS

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Abstract

With parallel coordinates (abbr. \parallel -cs) the perceptual barrier imposed by our 3-dimensional habitation is breached enabling the visualization of multidimensional problems. In this talk a panorama of highlights from the foundations to the most recent results, interlaced with applications and interactive demonstrations, will be intuitively developed. By learning to untangle **patterns** from \parallel -cs displays (Fig. 1, 2) a powerful knowledge discovery process has evolved. It is illustrated on real datasets together with guidelines for exploration and good query design.

Realizing that this approach is intrinsically limited (see Fig. 3 – left) leads to a deeper geometrical insight, the recognition of M -dimensional objects recursively from their $(M - 1)$ -dimensional subsets (Fig. 3 – right). Behind this striking cognitive success lies a special family of planes unique to \parallel -cs, the *superplanes*, whose points are represented by straight (rather than polygonal) lines. It emerges that **a hyperplane in N -dimensions is represented by $(N - 1)$ indexed points**. Points representing lines have two indices, those representing planes three indices and so on. In turn, this yields powerful geometrical algorithms (e.g. for intersections, containment, proximities) and applications including classification Fig. 4. The classifier's power is demonstrated by finding a rule recognizing hostile vehicles from their noise signature.

A smooth surface in 3-D is the envelope of its tangent planes each of which is represented by 2 points Fig. 6. As a result, a surface is represented by two planar regions and in N -dimensions by $(N - 1)$ regions. This is equivalent to representing the surface by its normal vectors, rather than projections as in standard surface descriptions. Developable surfaces are represented by curves Fig. 7 revealing the surfaces' characteristics. **Convex surfaces in any dimension** are recognized by the hyperbola-like (i.e. having two asymptotes) regions from just **one orientation** Fig. 5 – right, Fig. 8, Fig. 10 – right. Non-orientable surfaces (i.e. like the Möbius strip) yield stunning patterns Fig. 9 unlocking new geometrical insights. Non-convexities like folds, bumps, concavities and more are no longer hidden Fig. 10 – left and are detected from just one orientation. Evidently this representation is preferable for some applications even in 3-D.

The patterns persist in the presence of errors deforming in ways revealing the type and magnitude of the errors and that's good news for the applications. Processing the data directly, rather than the display, opens the way for the exploration of massive datasets. Only the results need be displayed; the patterns immensely concentrating the information (see again Fig. 3 - just the point is needed) without any display clutter. These are the “graphs” of multidimensional relations within the data. The challenge is to speed up the recursive algorithm, employing among others, intelligent agents to rapidly identify relational properties. We stand on the threshold of cracking the gridlock of multidimensional visualization.

The parallel coordinates methodology is used in collision avoidance and conflict resolution algorithms for air traffic control (3 USA patents), computer vision (USA patent), data mining (USA patent) for data exploration and automatic classification, optimization, process control and elsewhere.

EYE-CANDY

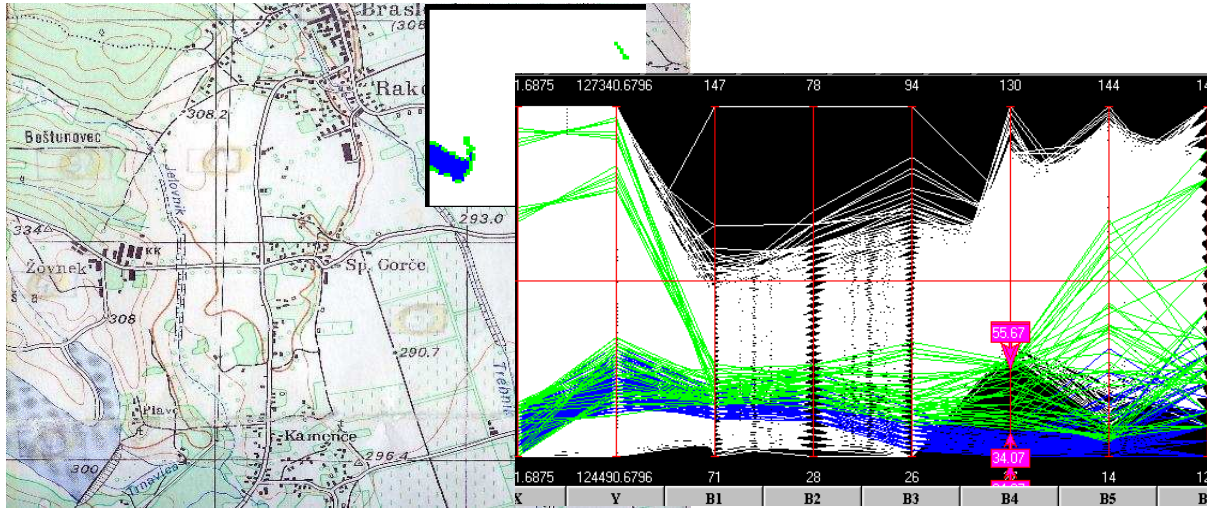


Figure 1: Exploratory Data Analysis, ground emissions measured by satellite over a region (left) are displayed on the right. In the middle, water (in blue) and the lake's edge (in green) are discovered by the indicated queries.

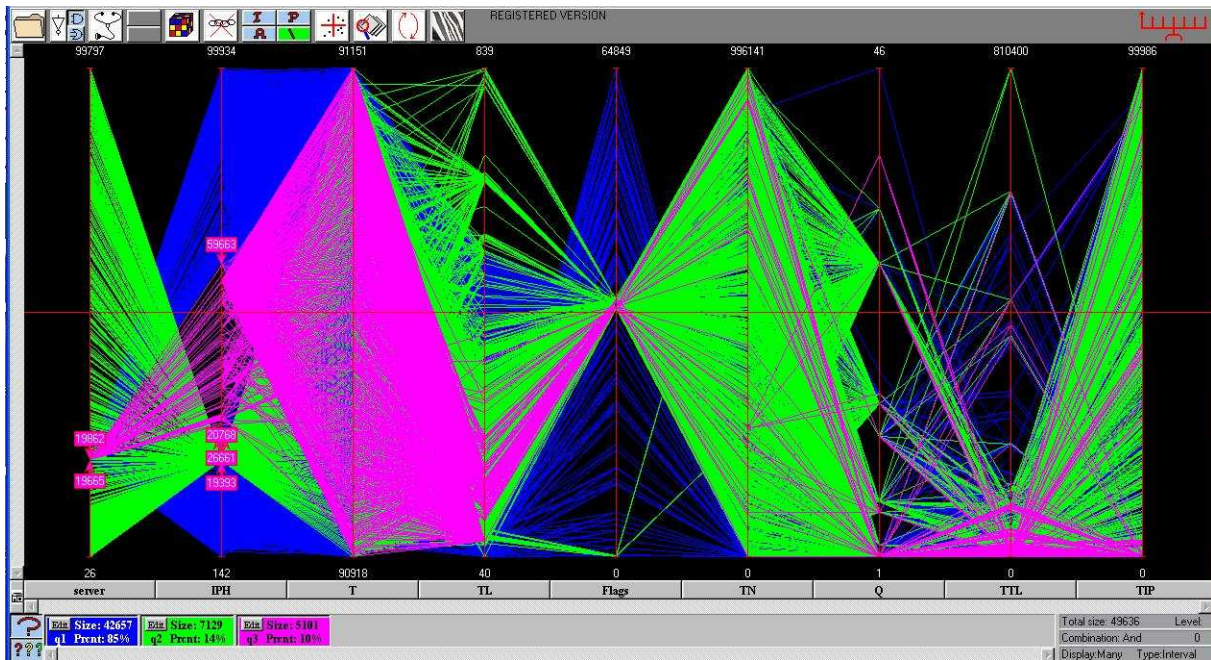


Figure 2: Detecting Network Intrusion from Internet Traffic Flow Data. Note the many-to-one relations.

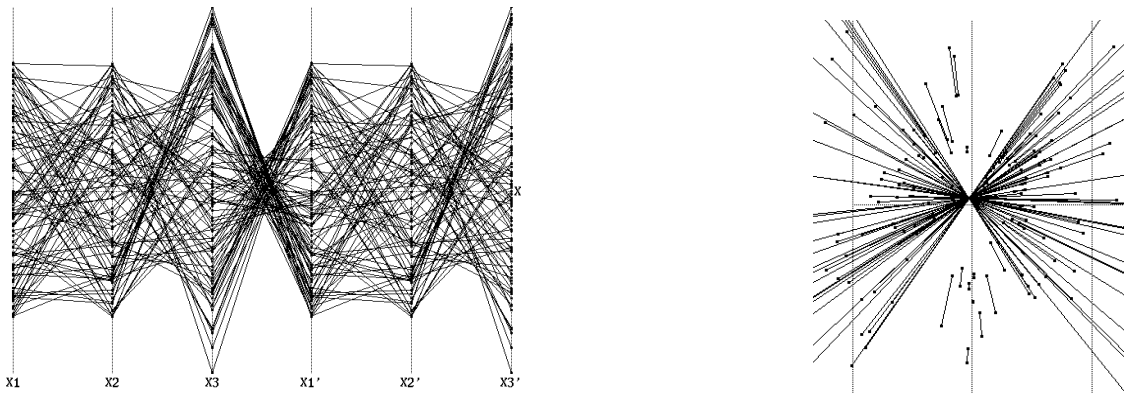


Figure 3: (left) Polygonal lines on the first 3 axes represent randomly chosen coplanar points. There is no discernible pattern. (right) Seeing coplanarity! Two points represent a line which is determined from the intersection (two points) of the corresponding two polygonal lines. **All** straight lines joining these pairs of points intersect. A plane is recognized from the representation of its *lines*. The *recursive* visualization generalizes to higher dimensions.

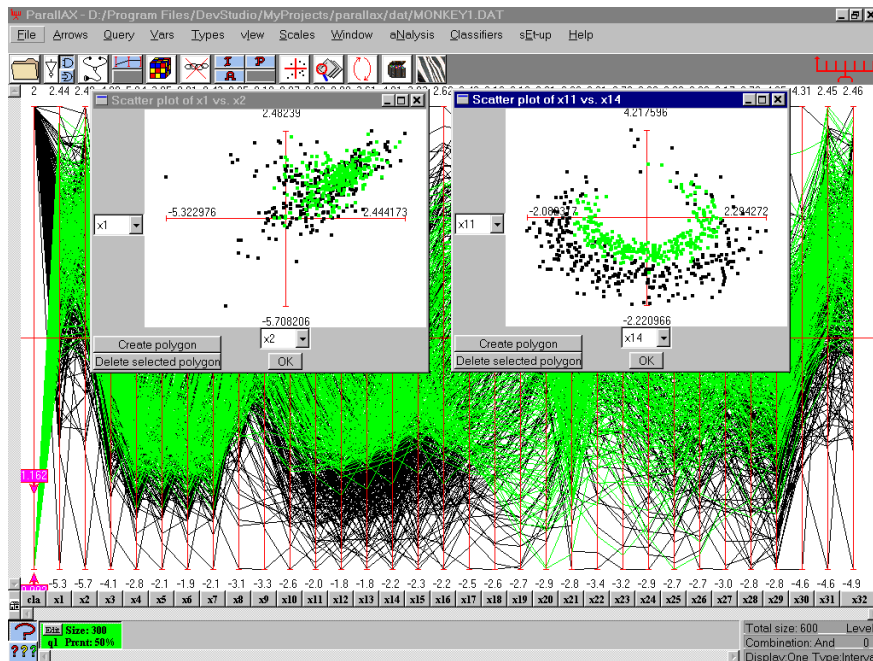


Figure 4: In the background is a dataset with 32 variables and 2 categories. On the left is the plot of the first two variables in the original order, on the right are the best two variables after classification. The algorithms discovers the best 9 variables (features) needed to describe the classification rule, with 4% error, and orders them according to their predictive power.

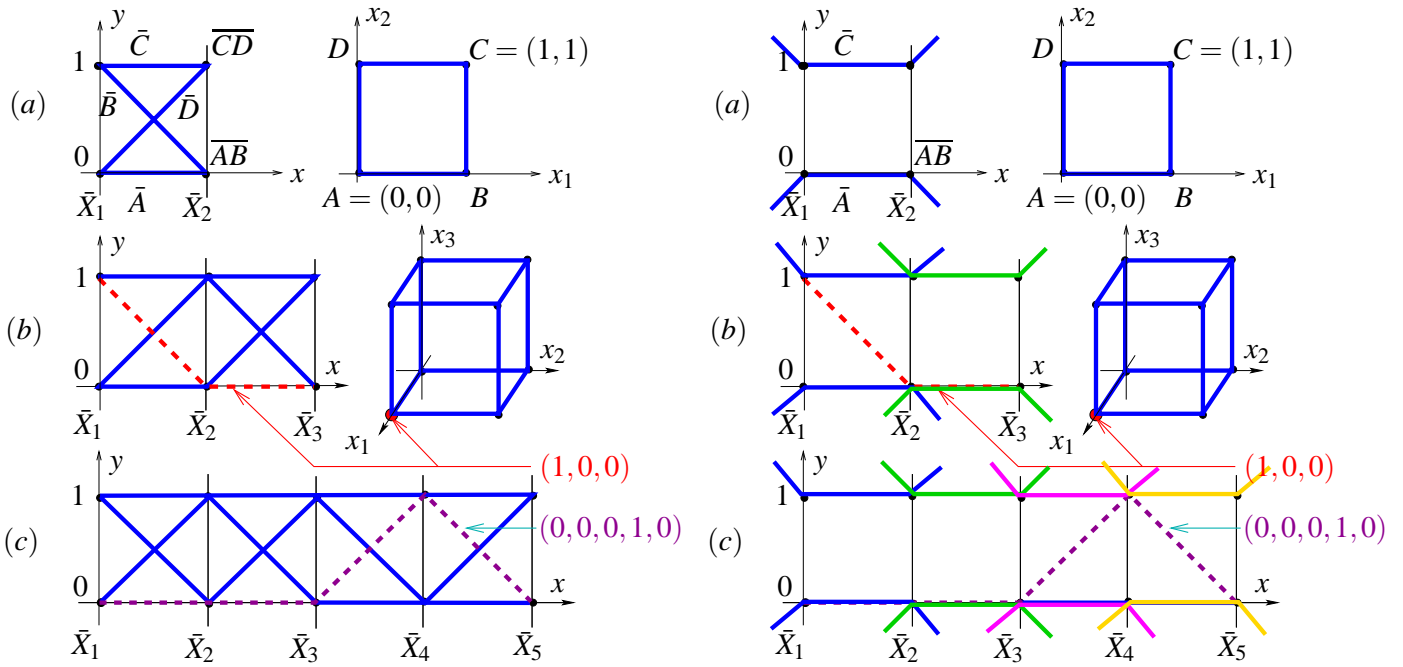


Figure 5: Square, cube and hypercube in 5-D on the left represented by their vertices and on the right by the tangent planes. Note the hyperbola-like (with 2 asymptotes) regions showing that the object is convex.

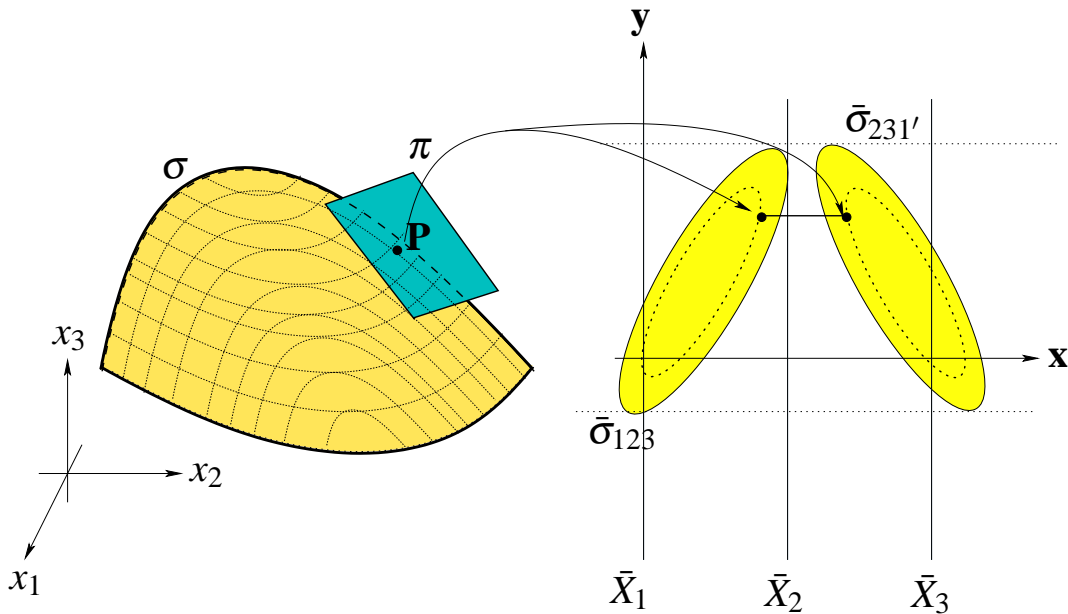


Figure 6: In 3-D a surface σ is represented by two linked planar regions $\bar{\sigma}_{123}$, $\bar{\sigma}_{231}'$. They consist of the pairs of points representing all its tangent planes. In N -dimensions a hypersurface is represented by $(N - 1)$ regions as the hypercube above.

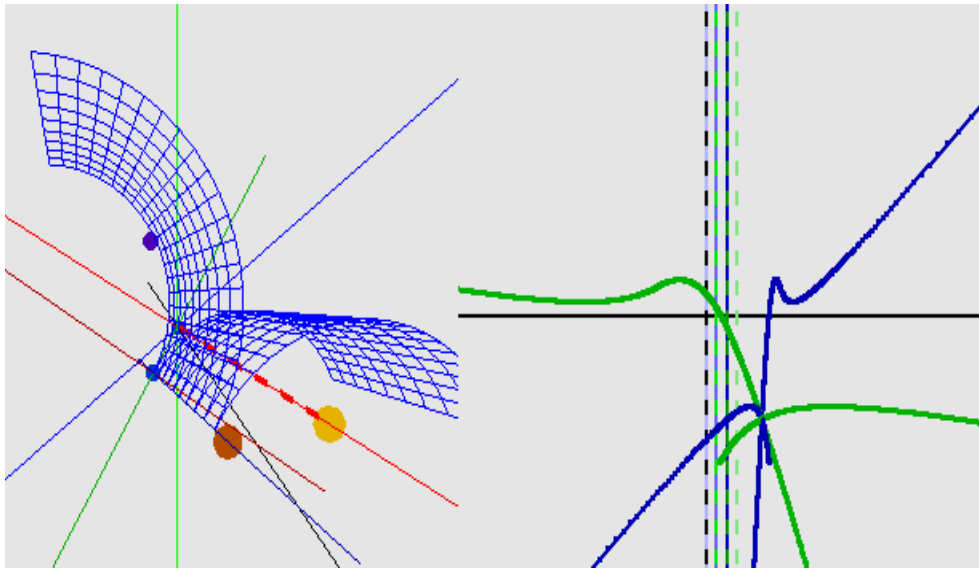


Figure 7: Developable surfaces are represented by curves. Note the two dualities *cusp* \leftrightarrow *inflection point* and *bitangent (tangent at two points) plane* \leftrightarrow *crossing point*. Three such curves represent the corresponding hypersurface in 4-D and so on.

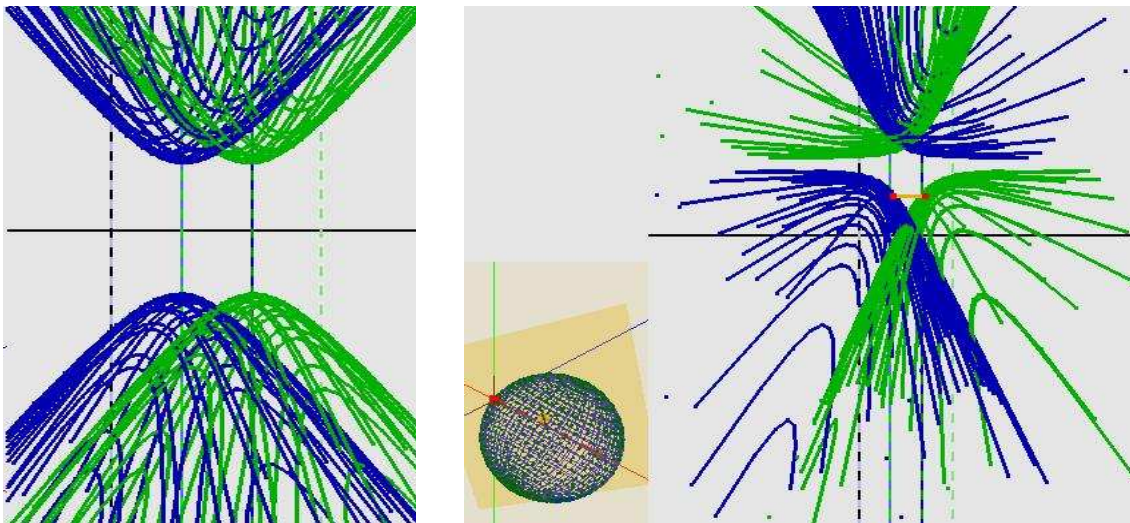


Figure 8: Representation of a sphere centered at the origin (left) and after a translation along the x_1 axis (right) causing the two hyperbolas to rotate in opposite directions illustrating the *rotation* \leftrightarrow *translation* duality. In N-D a sphere is represented by $(N - 1)$ such hyperbolic regions — pattern repeats as for the hypercube above.

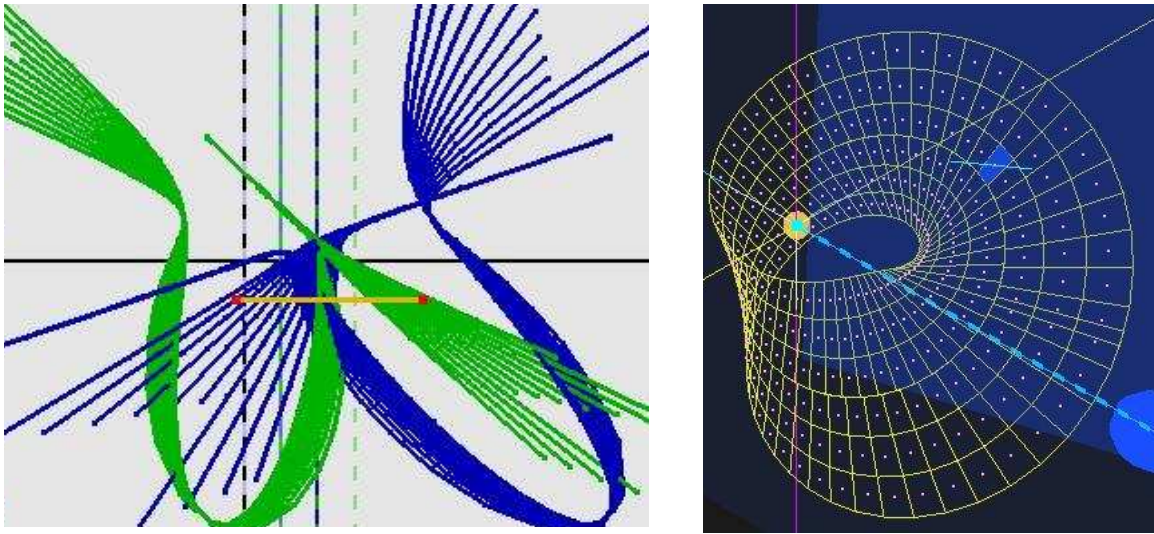


Figure 9: Möbius strip and its representation. Two cusps on the left represent the *twist* as an “inflection-point in 3-D” – see the duality in Fig 7. A tangent plane is represented by the indicated pair of points. The N -dimensional analogue of the of Möbius strip is represented by $(N - 1)$ such regions with cusps.

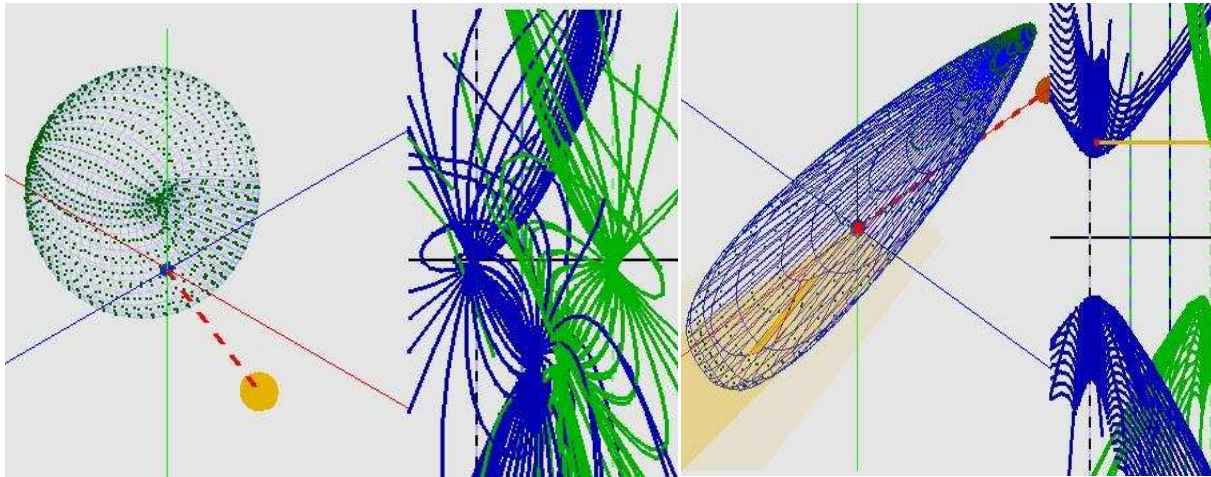


Figure 10: Representation of a surface with two *cusps* – only one is visible in the perspective. Each cusp in 3-D is mapped into a pair of “swirls”. The two pairs of swirls in the representation show that the surface has two cusps. On the right is a convex surface and its representation by hyperbola-like regions.