Problem 1:
Assume we have three random variables $X_1, X_2, X_3$, which are IID following a normal distribution with mean $\mu$ and variance $\sigma^2$. If we define two other random variables $Z_1, Z_2$ to be $Z_1 = X_1 + X_2$, $Z_2 = X_2 - X_3$. What are the distributions of $Z_1, Z_2$ respectively? What is the covariance of $Z_1$ and $Z_2$?

Hint: Linear combinations of normal r.v.’s are still normal but with different parameters.
Problem 2:
A probability density function $f$ satisfies:

$$f(x) = \begin{cases} \frac{\lambda e^{-\lambda x}}{1 - e^{-3\lambda}}, & 0 \leq x \leq 2 \\ 0, & \text{otherwise} \end{cases}$$

For some $\lambda \geq 0$. Suppose we have IID sample $X_1, X_2, \cdots, X_n$ follow this distribution $f$.

What is the MLE of the parameter $\lambda$?

**Hint:** It is not easy to give an explicit expression of $\hat{\lambda}_{\text{MLE}}$. Just give an equation that $\hat{\lambda}_{\text{MLE}}$ should satisfy.