

There are three questions, with a total of 40 points. Please start a new page for each question and do **not** use both sides.

1. [Total 3+3=6 points]

CSU officials are planning to audit 1386 new appointments to estimate the proportion p who have been incorrectly processed by the Payroll Office.

- (a) How large does the sample size need to be in order for $\frac{X}{n}$, the sample proportion, to have an 85% chance of lying within 0.04 of p ?
- (b) Past audits suggest that p will not be bigger than 0.10. Using this information, recalculate the sample size asked above.

2. [Total 6 points]

To study the metabolism of insects, researchers fed cockroaches measured amounts of a sugar solution. After 2, 5 and 10 hours, they dissected some of the cockroaches and measured the amount of sugar in various tissues. Five roaches fed the sugar D–glucose and dissected after 10 hours had the following amounts, in micrograms, of D–glucose in their hindguts: 55.95, 68.24, 52.73, 21.50 and 23.78. Assume the D–glucose content follows a normal distribution, construct a 95% confidence interval for the mean amount D–glucose in cockroach hindguts under these conditions.

3. [Total 8 points]

Suppose one observation Y_1 is drawn from the pdf

$$f_Y(y) = \begin{cases} \theta + 2(1 - \theta)y & \text{if } 0 \leq y \leq 1, \\ 0 & \text{otherwise,} \end{cases}$$

where $0 \leq \theta \leq 1$. For testing $H_0 : \theta \geq 0.75$ versus $H_1 : \theta < 0.75$, the following decision rule is to be employed: reject H_0 if $Y_1 \leq 0.20$. Find the power function of this test.

4. [Total 4+4=8 points]

A random sample of size 36 from a normal distribution having $\sigma = 4$ is observed.

- (a) Find the decision rule for testing $H_0 : \mu = 60$ versus $H_1 : \mu \neq 60$ with $\alpha = 0.05$.
- (b) Find $1 - \beta$ when the true μ is (i) 59 and (ii) 62.

5. [Total 4+4+4=12 points]

Let X_1, \dots, X_n be a random sample from the pdf $f_X(x; \theta) = \frac{\theta}{x^{\theta+1}}$, $x \geq 1$, $\theta > 0$.

- (a) Find an estimator for θ using maximum likelihood.
- (b) Find the form of the GLRT for testing $H_0 : \theta = 1$ versus $H_1 : \theta \neq 1$.
- (c) Let $p = \prod_{i=1}^n X_i$. Prove or disprove that the decision rule for part (b) has the form: reject if $\log p - n \log \log p > k$ for some constant k .

— End of Midterm II —