Basic Time Series

R code at http://www.stat.columbia.edu/~madigan/W2025/code/timeseries.R

Linear Filtering

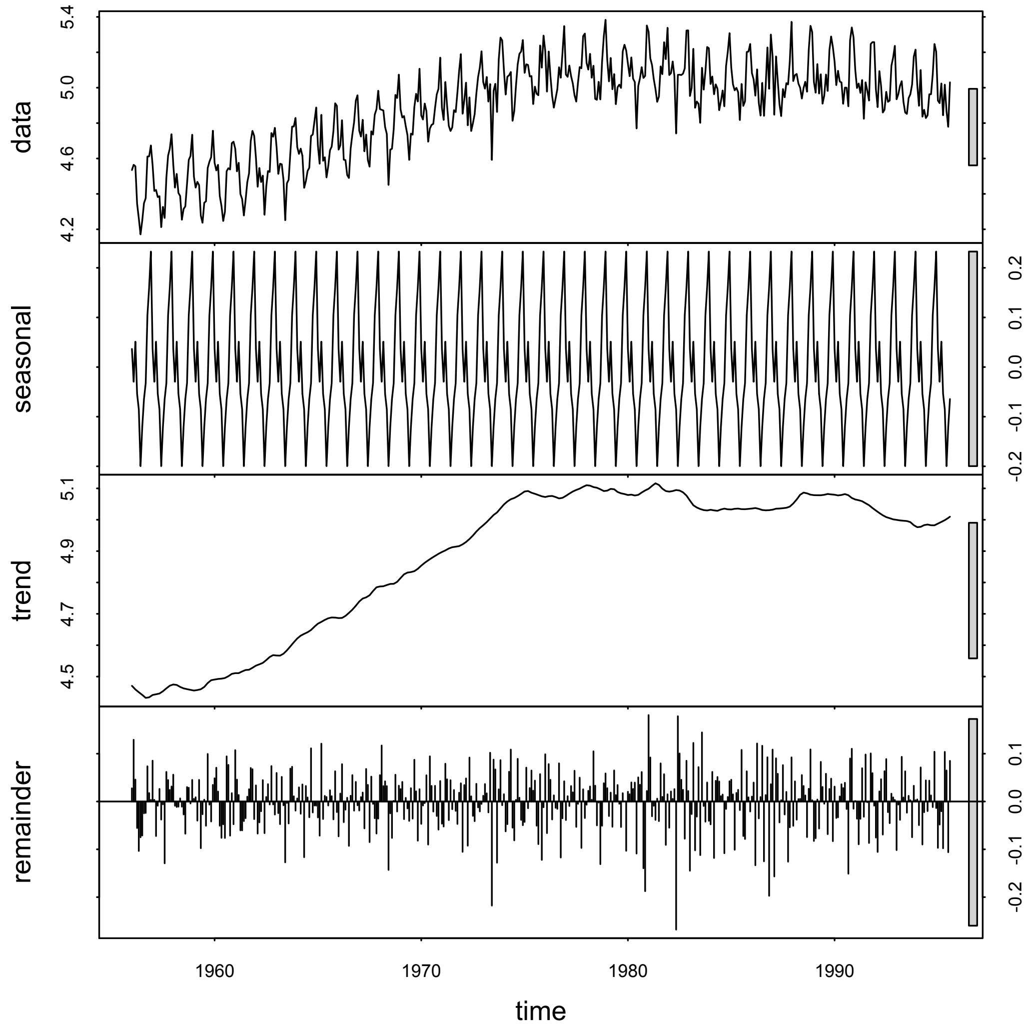
$$T_t = \sum_{i=-\infty}^{\infty} \lambda_i X_{t+i}$$

Moving average:

$$T_t = \frac{1}{2a+1} \sum_{i=-a}^{a} X_{t+i}$$

- $a = 2 : \lambda_i = \{\frac{1}{5}, \frac{1}{5}, \frac{1}{5}, \frac{1}{5}, \frac{1}{5}\}$
- $a = 12 : \lambda_i = \underbrace{\{\frac{1}{25}, \ldots, \frac{1}{25}\}}_{25 \text{ times}}$
- $a = 40 : \lambda_i = \underbrace{\{\frac{1}{81}, \ldots, \frac{1}{81}\}}_{81 \text{ times}}$

Decomposition of Time Series



Exponential Smoothing

- Obvious forecasting model:

$$\hat{x}_{(t=\tau)}(1) = \lambda_0 \cdot x_\tau + \lambda_1 \cdot x_{\tau-1} + \ldots$$

- Could use geometric weights:

$$\lambda_i = \alpha(1-\alpha)^i \qquad ; \quad 0 < \alpha < 1$$

$$\hat{x}_{(t=\tau)}(1) = \alpha \cdot x_\tau + \alpha(1-\alpha) \cdot x_{\tau-1} + \alpha(1-\alpha)^2 \cdot x_{\tau-2} + \ldots$$

weights add to one

Holt-Winters

- exponential smoothing with trend and seasonal variation

- three parameters: α (for the level), β (for the trend) and γ (for the seasonal variation)

Stationarity

- Properties of estimators depend on whether series is stationary or not
- Time series y_t is stationary if its probability density function does not depend on time i.e. pdf of $(y_s, y_{s+1}, y_{s+2}, .. y_{s+t})$ does not depend on s
- Implies:
  - $E(y_t)$ does not depend on t
  - $Var(y_t)$ does not depend on t
  - $Cov(y_t, y_{t+s})$ depends on s and not t

Simplest Stationary Process

$$y_t = \alpha_0 + \varepsilon_t$$

- Where ε_t is 'white noise' – iid with mean 0 and variance σ2
- Simple to check that:
  - $E(y_t)=\alpha_0$
  - $Var(y_t)= \sigma^2$
  - $Cov(y_t,y_{t-s})=0$
- Implies y_t is 'white noise' – unlikely for most economic time series

http://econ.lse.ac.uk/~amanning/courses/ec402/UnivariateTimeSeries.ppt

First-Order Autoregressive Process AR(1)

$$y_t = \alpha_0 + \alpha_1 y_{t-1} + \varepsilon_t$$

$$E\left(y_t \middle| y_{t-1}, \ldots, y_0\right) = \alpha_0 + \alpha_1 y_{t-1}$$

$|\alpha_1| < 1$ – this is the condition for stationarity of AR(1) process

More General Auto-Regressive Processes

- AR(p) can be written as:

$$y_t = \alpha_0 + \sum_{i=1}^{p} \alpha_i y_{t-i} + \varepsilon_t$$

- Necessary condition for stationarity is:

$$-1 < \sum_{i=1}^{p} \alpha_i < 1$$

Moving-Average Processes

- Most common alternative to an AR process – MA(1) can be written as:

$$y_t = \alpha_0 + \varepsilon_t + \theta\varepsilon_{t-1}$$

- MA process will always be stationary

MA(q) Process

$$y_t = \alpha_0 + \varepsilon_t + \sum_{i=1}^{q} \theta_i \varepsilon_{t-i}$$

- Will always be stationary
- Covariances between two observations zero if more than q periods apart

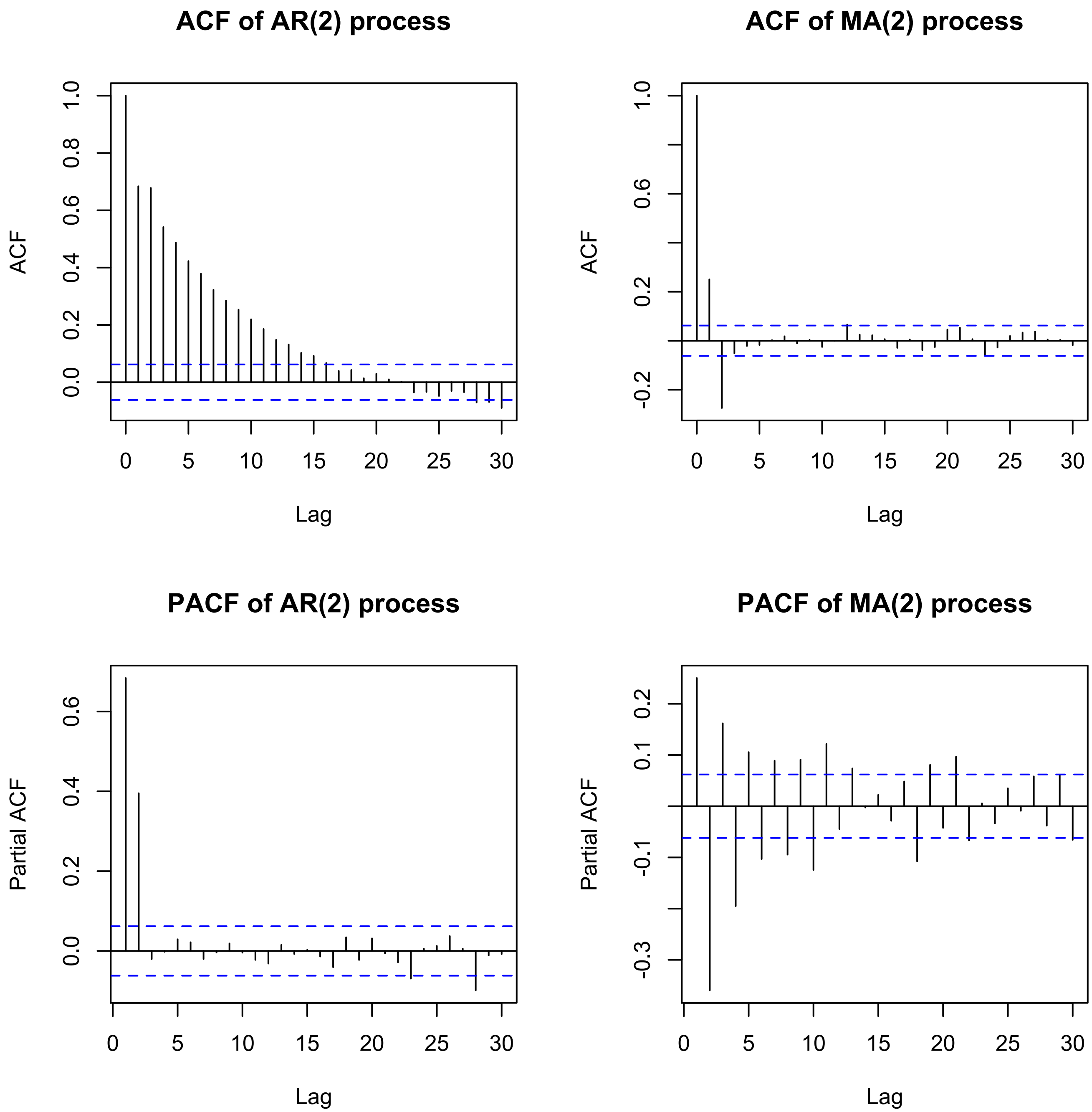
ARMA Processes

- Time series might have both AR and MA components
- ARMA(p,q) can be written as:

$$y_t = \alpha_0 + \sum_{i=1}^{p} \alpha_i y_{t-i} + \varepsilon_t + \sum_{i=1}^{q} \theta_i \varepsilon_{t-i}$$

ARIMA/Box-Jenkins Modeling

- Model identification
- Parameter estimation
- Diagnostic checking


ACF of AR(2) process
ACF
0.0 0.2 0.4 0.6 0.8 1.0
0 5 10 15 20 25 30
Lag
ACF of MA(2) process
ACF
-0.2 0.2 0.6 1.0
0 5 10 15 20 25 30
Lag
PACF of AR(2) process
Partial ACF
0.0 0.2 0.4 0.6
0 5 10 15 20 25 30
Lag
PACF of MA(2) process
Partial ACF
-0.3 -0.1 0.1 0.2
0 5 10 15 20 25 30
Lag