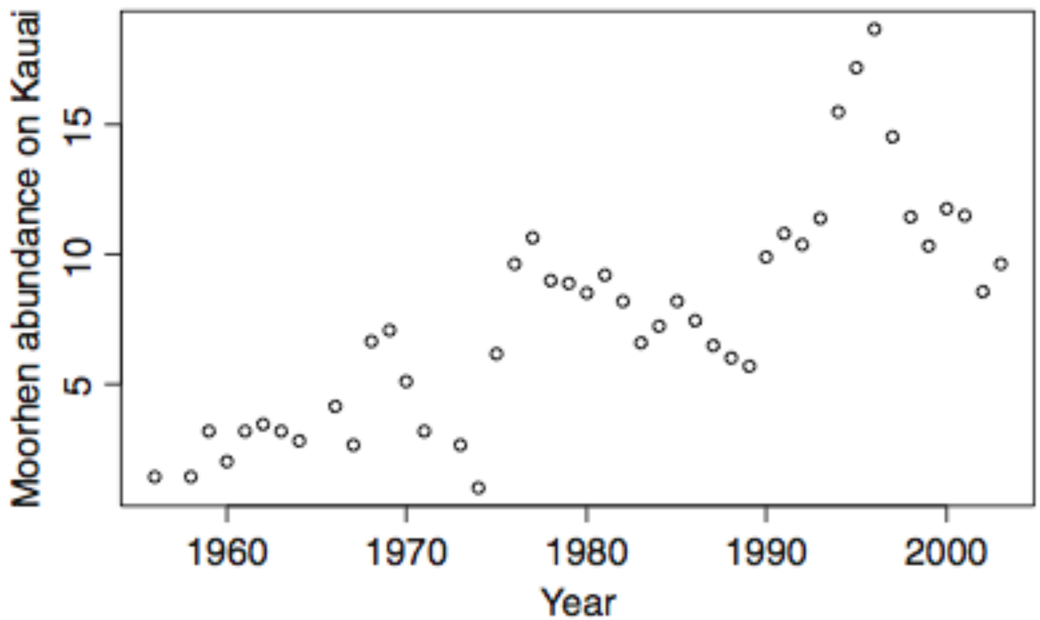
Temporal Correlation in Regression

Zuur et al., Chapter 6

Reed et al. (2007) analysed abundances of three bird species measured at three islands in Hawaii. The data were annual abundances from 1956 to 2003. Here, we use one of these time series, moorhen abundance on the island of Kauai, to illustrate how to deal with violation of independence.



```
> library(AED); data(Hawaii)
> Hawaii$Birds <- sqrt(Hawaii$Moorhen.Kauai)
> plot(Hawaii$Year, Hawaii$Birds, xlab = "Year",
    ylab = "Moorhen abundance on Kauai")
```

$$\text{Birds}_s = \alpha + \beta_1 \times \text{Rainfall}_s + \beta_2 \times \text{Year}_s + \varepsilon_s$$

$$\varepsilon_s \sim N(0, \sigma^2)$$

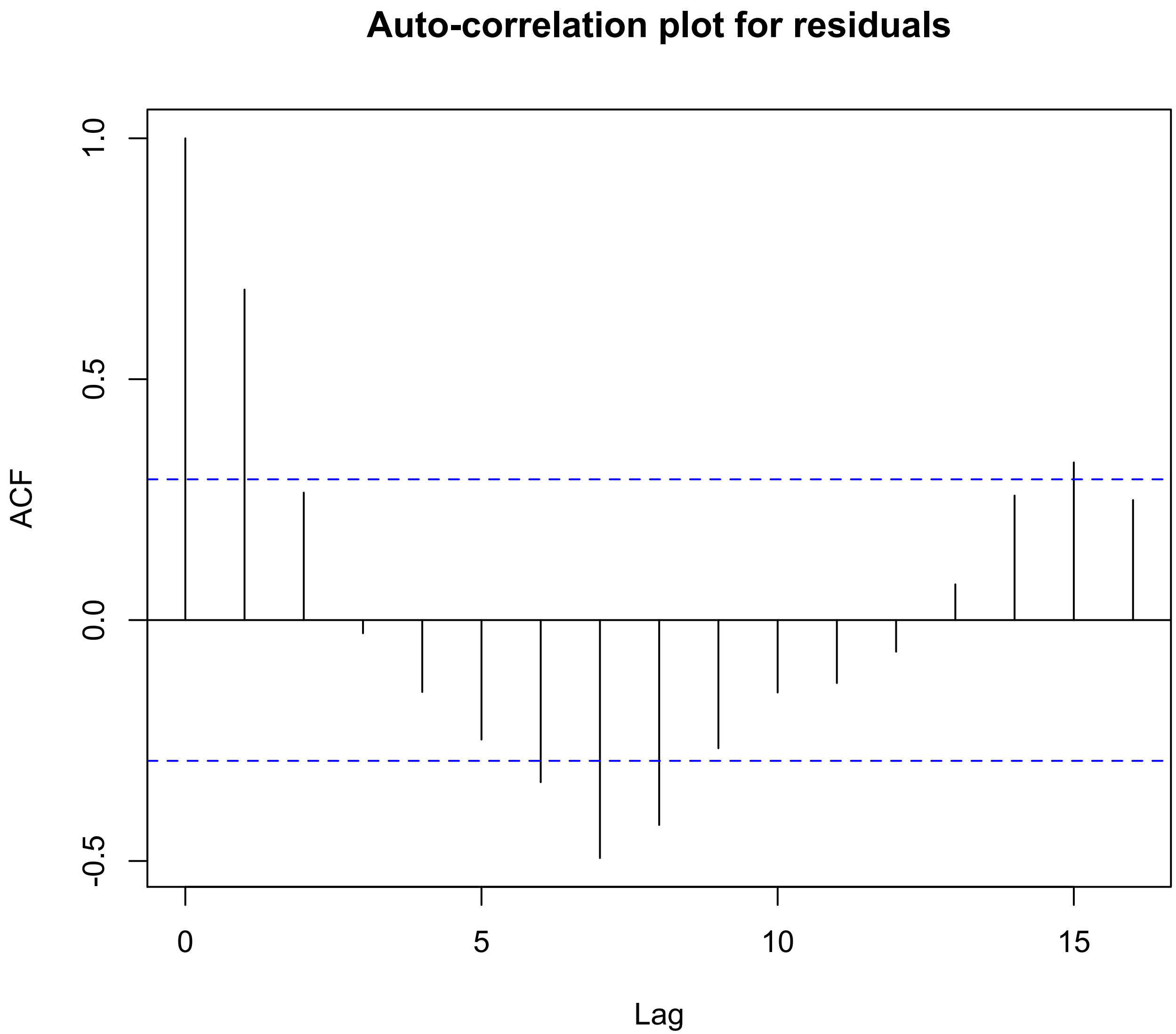
$$\text{cov}(\varepsilon_s, \varepsilon_t) = \begin{cases} \sigma^2 & \text{if } s = t \\ 0 & \text{else} \end{cases}$$

```
> library(nlme)
> M0 <- gls(Birds ~ Rainfall + Year,
            na.action = na.omit, data = Hawaii)
> summary(M0)
```

```
I1 <- !is.na(Hawaii$Birds)
E <- residuals(M0, type = "normalized")
plot(Hawaii[I1,]$Year,E,type="l")
acf(E, main = "Auto-correlation plot for residuals")
```

```
M11<-lm(time~dist+climb,data=hills)
acf(residuals(M11))
```

E
2
1
0
-1
-2
1960
1970
1980
1990
2000
Hawaii[I1,]$Year



Auto-correlation plot for residuals
ACF
1.0
0.5
0.0
-0.5
0
5
10
15
Lag

could model the entire covariance matrix:

$$\mathbf{V} = \mathrm{cov}(\boldsymbol{\varepsilon}) = \begin{pmatrix} \mathrm{var}(\varepsilon_{1958}) & & & & \\ \mathrm{cov}(\varepsilon_{1959}, \varepsilon_{1958}) & \mathrm{var}(\varepsilon_{1959}) & & & \\ \mathrm{cov}(\varepsilon_{1960}, \varepsilon_{1958}) & \mathrm{cov}(\varepsilon_{1960}, \varepsilon_{1959}) & \ddots & & \\ \vdots & \vdots & \cdots & \ddots & \\ \mathrm{cov}(\varepsilon_{2003}, \varepsilon_{1958}) & \mathrm{cov}(\varepsilon_{2003}, \varepsilon_{1959}) & \cdots & \mathrm{cov}(\varepsilon_{2003}, \varepsilon_{2002}) & \mathrm{var}(\varepsilon_{2003}) \end{pmatrix}$$

or could make simplifying assumptions:

$$\mathrm{cor}(\boldsymbol{\varepsilon}) = \begin{pmatrix} 1 & \rho & \rho & \cdots & \rho \\ \rho & 1 & \rho & \cdots & \rho \\ \rho & \rho & \ddots & \vdots & \vdots \\ \vdots & \vdots & \cdots & \ddots & \rho \\ \rho & \rho & \cdots & \rho & 1 \end{pmatrix}$$

```
> M1 <- gls(Birds ~ Rainfall + Year,
            na.action = na.omit, data = Hawaii ,
            correlation = corCompSymm(form =~ Year))
```

AR(1)

$$\varepsilon_s = \rho \varepsilon_{s-1} + \eta_s$$

$$\text{cor}(\varepsilon_s, \varepsilon_t) = \begin{cases} 1 & \text{if } s = t \\ \rho^{|t-s|} & \text{else} \end{cases}$$

$$\mathrm{cor}(\boldsymbol{\varepsilon}) = \begin{pmatrix} 1 & \rho & \rho^2 & \rho^3 & \cdots & \rho^{57} \\ \rho & 1 & \rho & \ddots & \ddots & \vdots \\ \rho^2 & \rho & 1 & \ddots & \rho^2 & \rho^3 \\ \rho^3 & \rho^2 & \rho & \ddots & \rho & \rho^2 \\ \vdots & \ddots & \ddots & \ddots & 1 & \rho \\ \rho^{57} & \cdots & \rho^3 & \rho^2 & \rho & 1 \end{pmatrix}$$

```
> M2 <- gls(Birds ~ Rainfall + Year,
            na.action = na.omit, data = Hawaii,
            correlation = corAR1(form =~ Year))
> summary(M2)
```

AIC(M0,M1,M2)