Bayesian Graphical Models for Location Determination

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The Problem

- Estimate the physical location of a wireless terminal/user in an enterprise
  - Radio wireless communication network, specifically, 802.11-based
Example Applications

• Use the closest resource, e.g., printing to the closest printer

• Security: in/out of a building

• Emergency 911 services

• Privileges based on security regions (e.g., in a manufacturing plant)

• Equipment location (e.g., in a hospital)

• Mobile robotics

• Museum information systems
Automakers go wireless to find cars in plants
Physical Features Available for Use

- Received Signal Strength (RSS) from multiple access points
- Angles of arrival
- Time deltas of arrival
- Which access point (AP) you are associated with

- We use RSS and AP association
  - RSS is the only reasonable estimate with current commercial hardware
Known Properties of Signal Strength

- Signal strength at a location is known to vary as a log-normal distribution with some environment-dependent $\sigma$

- Variation caused by people, appliances, climate, etc.

- The Physics: signal strength ($SS; \text{in dB}$) is known to decay with distance ($d$) as $SS = k_1 + k_2 \log d$
Location Determination via Statistical Modeling

- Data collection is slow, expensive ("profiling")
- "Productization"
- Either the access points or the wireless devices can gather the data
- Focus on predictive accuracy
Prior Work

- Take signal strength measures at many points in the site and do a closest match to these points in signal strength vector space. [e.g. Microsoft’s RADAR system]

- Take signal strength measures at many points in the site and build a multivariate regression model to predict location (e.g., Tirri’s group in Finland)

- Some work has utilized wall thickness and materials
Krishnan et al. Results
Infocom 2004

– Smoothed signal map per access point + nearest neighbor
- Best result: mean error 2.57 meters (90% below 4.52 meters) obtained with the probabilistic histogram method with tracking.
- Surprisingly robust with respect to the amount of training data.
Probabilistic Graphical Models

- Graphical model = picture of some conditional independence assumptions

- For example, $D_1$ is conditionally independent of $D_3$ given $X$
Markov Properties for Acyclic Directed Graphs
(Bayesian Networks)

(Global) S separates A from B in $G_{an(A,B,S)} \Rightarrow A \perp B \mid S$

(Local) $\alpha \perp nd(\alpha) \setminus pa(\alpha) \mid pa(\alpha)$

(Factorization) $f(x) = \prod f(x_v \mid x_{pa(v)})$

\[ p(X,D_1,D_2,D_3,S_1,S_2) \]
\[ = p(X) p(D_1 \mid X) p(D_2 \mid X) p(D_3 \mid X) \]
\[ p(S_1 \mid D_1,D_2) p(S_2 \mid D_2) \]
Monte Carlo Methods and Graphical Models

Simple Monte Carlo: Sample in turn from

\[ p(X), p(D_1 | X), p(D_2 | X), p(D_3 | X), \]

\[ p(S_1 | D_1, D_2), \text{ and } p(S_2 | D_2) \]

Gibbs Sampling: Sample in turn from

\[ p(X \mid D_1, D_2, D_3, S_1, S_2) \]

\[ p(D_1 \mid X, D_2, D_3, S_1, S_2) \]

\[ \ldots \]

\[ p(S_2 \mid X, D_1, D_2, D_3, S_1) \]


BUGS/WinBUGS automates this via adaptive rejection sampling and slice sampling
Full Conditionals from the Graphical Model

**Incorporating Data, etc.** Suppose the $D$’s were observed. Then sample from:

\[
\begin{align*}
    p(X \mid D_1,D_2,D_3,S_1,S_2) \\
    p(S_1 \mid X, D_1,D_2,D_3, S_2) \\
    p(S_2 \mid X, D_1,D_2,D_3,S_1)
\end{align*}
\]
Full Conditionals from the Graphical Model

**Incorporating Data, etc.** Suppose the $D$’s were observed. Then sample from:

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\[ p(S_2 \mid X, D_1, D_2, D_3, S_1) \]

**Bayesian Analysis.** Treat “parameters” the same as everything else.
• **BR has 5 APs**, site dimension: 225 ft X 144 ft
  
  - 259 blue (corridor) data points taken earlier
Bayesian Graphical Model Approach

$X, Y \sim \text{unif}$

$D_i(X, Y) = \text{distance to the } i\text{th access point}$

$S_i \sim N(b_{i0} + b_{i1} \log D_i, \sigma_i^2), \; i = 1,...,5$
Plate Notation

\[ X_i \rightarrow D_{ij} \rightarrow S_{ij} \rightarrow b_{0j} \rightarrow b_{1j} \]

\[ Y_i \rightarrow D_{ij} \rightarrow S_{ij} \rightarrow b_{0j} \rightarrow b_{1j} \]

\[ i = 1, \ldots, n \]

\[ j = 1, \ldots, 5 \]
SmoothNN (S) versus Bayesian (B) Model, Error in Feet
Hierarchical Model
Hierarchical Model

\[ X_i \rightarrow D_{ij} \rightarrow S_{ij} \]

\[ i = 1, \ldots, n \]

\[ j = 1, \ldots, 5 \]

\[ Y_i \rightarrow D_{ij} \rightarrow S_{ij} \]
Simple Bayesian (B) versus Hierarchical Bayesian (H) Model, Error in Feet
Pros and Cons

• Bayesian model produces a predictive distribution for location

• MCMC can be slow

• Difficult to automate MCMC (convergence issues)

• Perl-WinBUGS (perl selects training and test data, writes the WinBUGS code, calls WinBUGS, parses the output file)
What if we had no locations in the training data?
Results with No Locations: Simple (S), Hierarchical (H), Error in Feet
Zero Profiling?

- Simple sniffing devices can gather signal strength vectors from available WiFi devices
- Can do this repeatedly
- Locations of the Access Points
Why does this work?

- Prior knowledge about distance-signal strength
- Prior knowledge that access points behave similarly
- Estimating several locations simultaneously
Corridor Effects
Results for N=20, no locations

<table>
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<th>corridor main effect</th>
<th>corridor-distance interaction</th>
<th>average error</th>
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<td>1</td>
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with mildly informative prior on the distance main effect…

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<tr>
<td>1</td>
<td>1</td>
<td>15.9</td>
</tr>
</tbody>
</table>
Corridor Effect: None (N), Main (M), Interaction (I), Both (B), Error in Feet

- N=5
- N=10
- N=20
- N=50
- N=100
- N=254

- CA Down Data
- CA Up Data

Corridor Effect: None (N), Main (M), Interaction (I), Both (B), Error in Feet
Discussion

• Informative priors
• Convenience and flexibility of the graphical modeling framework
• Censoring (30% of the signal strength measurements)
• Repeated measurements & normal error model
• Tracking

• Machine learning-style experimentation is clumsy with perl-WinBUGS
• BR has 5 APs, site dimension: 225 ft X 144 ft
  – 259 blue (corridor) data points taken earlier
Prior Work

• Use physical characteristics of signal strength propagation and build a model augmented with a wall attenuation factor

• Needs detailed (wall) map of the building; model portability needs to be determined