Bell-Shaped Curves and Other Shapes

Chapter 8

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Thought Question 1:

The **heights** of adult women
in the United States follow,
at least **approximately,**
a bell-shaped curve.

What do you think this means?

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Thought Question 2:

What does it mean to say
that a man's weight
is in the **30^{th} percentile**
for all adult males?

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Thought Question 3:

A "**standardized score**" is simply the **number of standard deviations an individual falls above or below the mean for the whole group**.

Male heights have a mean of 70 inches and a standard deviation of 3 inches. Female heights have a mean of 65 inches and a standard deviation of 2 ½ inches. Thus, a man who is 73 inches tall has a standardized score of 1.

What is the standardized score corresponding to your own height?

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Thought Question 4:

Data sets consisting of physical measurements (heights, weights, lengths of bones, and so on) for adults of the same species and sex tend to follow a similar pattern.

The pattern is that most individuals are clumped around the average, with numbers decreasing the farther values are from the average in either direction.

Describe **what shape** a histogram of such measurements would have.

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8.1 Populations, Frequency Curves, and Proportions

Move from pictures and shapes of a set of data to …

Pictures and shapes for populations of measurements.

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Frequency Curves

Smoothed-out histogram by connecting tops of rectangles with smooth curve.

Frequency curve for population of British male heights.

The measurements follow a **normal** distribution (or a bell-shaped or Gaussian curve).

Note: Height of curve set so area under entire curve is 1.

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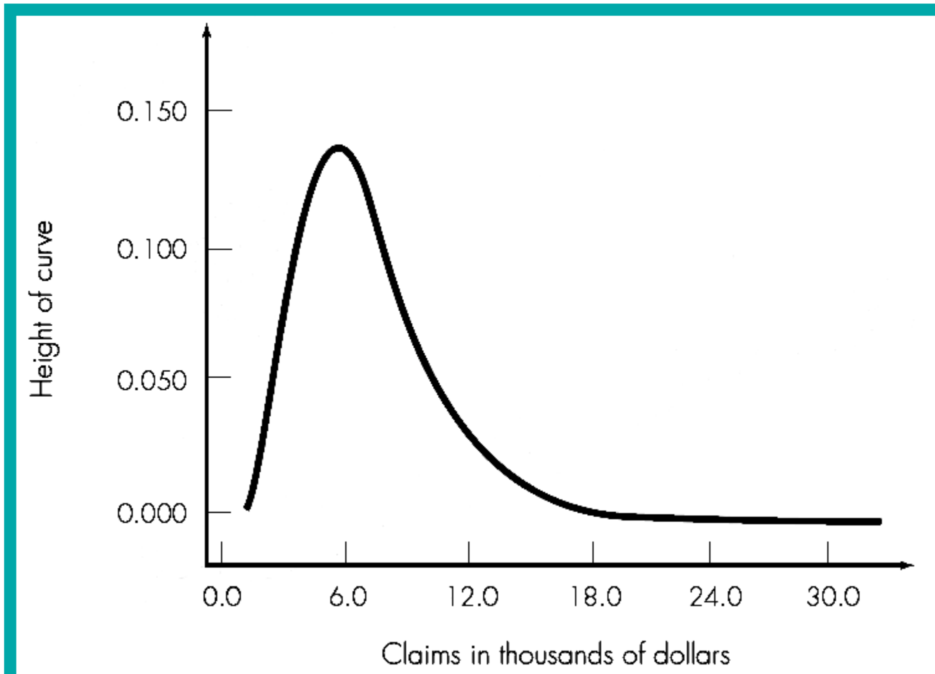
Frequency Curves

Not all frequency curves are bell-shaped!

Frequency curve for population of dollar amounts of car insurance damage claims.



The measurements follow a **right skewed** distribution. Majority of claims were below $5,000, but there were occasionally a few extremely high claims.

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Proportions

Recall: Total area under frequency curve = 1 for 100%

Key: ***Proportion*** of population of measurements falling in a certain range = ***area*** under curve over that range.

Mean British Height is 68.25 inches. Area to the right of the mean is 0.50. So ***about half of all British men are 68.25 inches or taller***.

Tables will provide other areas under normal curves.

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8.2 The Pervasiveness of Normal Curves

Many populations of measurements follow approximately a normal curve:

- Physical measurements within a homogeneous population – heights of male adults.
- Standard academic tests given to a large group – SAT scores.

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8.3 Percentiles and Standardized Scores

Your percentile = the percentage of the population that falls *below* you.

Finding percentiles for normal curves requires:

- Your **own value**.
- The **mean** for the population of values.
- The **standard deviation** for the population.

Then any bell curve can be ***standardized*** so one table can be used to find percentiles.

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Standardized Scores

Standardized Score (standard score or *z*-score):

$$\frac{\textbf{observed value} - \textbf{mean}}{\textbf{standard deviation}}$$

IQ scores have a **normal** distribution with a **mean of 100** and a **standard deviation of 16**.

- Suppose your IQ **score was 116**.
- Standardized score = (116 – 100)/16 = +1
- Your IQ is 1 standard deviation ***above*** the mean.

- Suppose your IQ **score was 84**.
- Standardized score = (84 – 100)/16 = –1
- Your IQ is 1 standard deviation ***below*** the mean.

A **normal curve** with **mean = 0** and **standard deviation = 1** is called a **standard normal curve**.

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Table 8.1: Proportions and Percentiles for Standard Normal Scores

Standard Score, *z*	Proportion Below *z*	Percentile	Standard Score, *z*	Proportion Below *z*	Percentile
-6.00	0.000000001	0.0000001	0.03	0.51	51
-5.20	0.0000001	0.00001	0.05	0.52	52
-4.26	0.00001	0.001	0.08	0.53	53
-3.00	0.0013	0.13	0.10	0.54	54
:	:	:	:	:	:
-1.00	0.16	16	0.58	0.72	72
:	:	:	:	:	:
-0.58	0.28	28	1.00	0.84	84
:	:	:	:	:	:
0.00	0.50	50	6.00	0.999999999	99.9999999

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Finding a Percentile from an observed value:

1. Find the **standardized score** = (observed value – mean)/s.d., where s.d. = standard deviation.
   Don't forget to keep the plus or minus sign.
2. Look up the percentile in **Table 8.1**.

- Suppose your IQ **score was 116**.
- Standardized score = (116 – 100)/16 = +1
- Your IQ is 1 standard deviation ***above*** the mean.
- From Table 8.1 you would be at the **84th percentile**.
- Your IQ would be higher than that of 84% of the population.

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Finding an Observed Value from a Percentile:

1. Look up the percentile in **Table 8.1** and find the corresponding standardized score.
2. Compute **observed value** = mean +(standardized score)(s.d.), where s.d. = standard deviation.

Example 1: Tragically Low IQ

"Jury urges mercy for mother who killed baby. …
The mother had an IQ lower than 98 percent of the population."
(Scotsman, March 8, 1994,p. 2)

- Mother was in the 2^{nd} percentile.
- Table 8.1 gives her standardized score = –2.05, or 2.05 standard deviations below the mean of 100.
- Her IQ = 100 + (–2.05)(16) = 100 – 32.8 = 67.2 or about 67.

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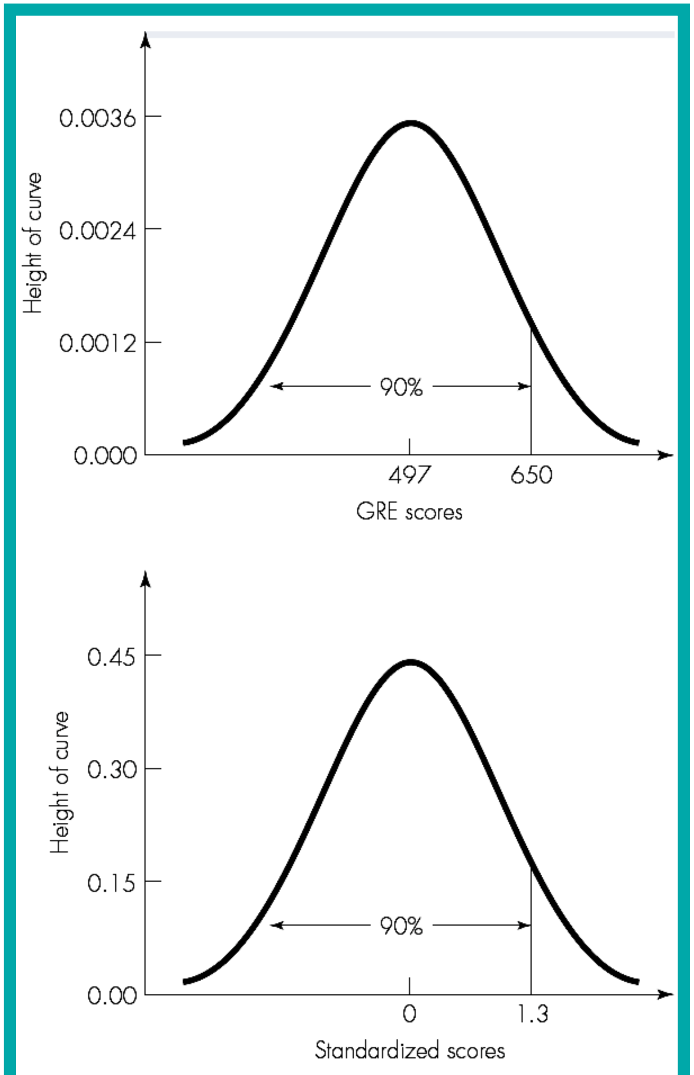
Example 2: Calibrating Your GRE Score

GRE Exams between 10/1/89 and 9/30/92 had **mean verbal score of 497 and a standard deviation of 115**. (ETS, 1993)

Suppose your **score was 650** and scores were bell-shaped.

- Standardized score = (650 – 497)/115 = +1.33.
- Table 8.1, $z = 1.33$ is between the 90th and 91st percentile.
- Your score was higher than about 90% of the population.



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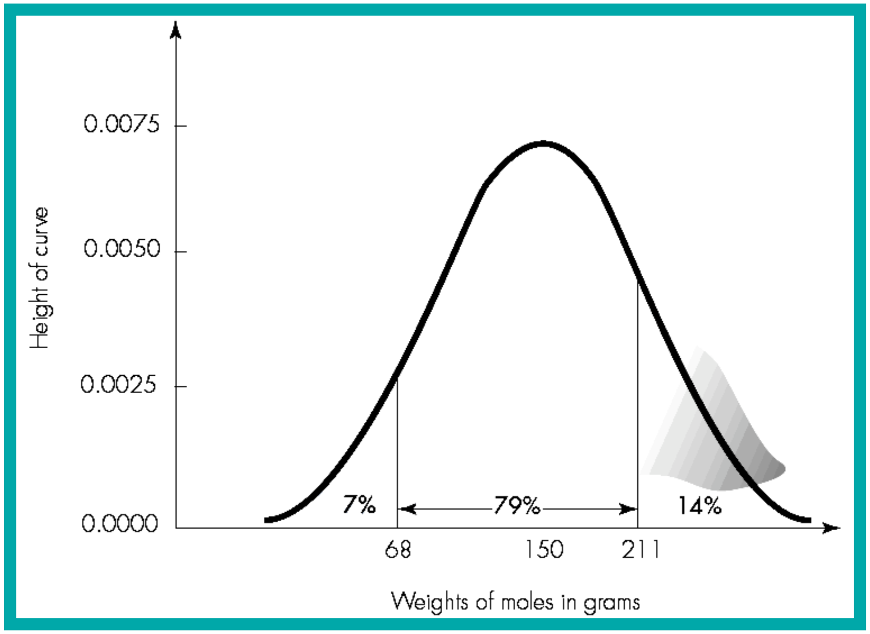
Example 3: Removing Moles

Company ***Molegon***: remove unwanted moles from gardens.

Weights of moles are approximately normal with a mean of 150 grams and a standard deviation of 56 grams.

Only moles between 68 and 211 grams can be legally caught.

- Standardized score = (68 – 150)/56 = –1.46, and Standardized score = (211 – 150)/56 = +1.09.
- Table 8.1: 86% weigh 211 or less; 7% weigh 68 or less.
- About 86% – 7% = 79% are within the legal limits.



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8.4 *z*-Scores and Familiar Intervals

Empirical Rule

For any **normal curve**, approximately …

- **68%** of the values fall within **1 standard deviation** of the mean in either direction
- **95%** of the values fall within **2 standard deviations** of the mean in either direction
- **99.7%** of the values fall within **3 standard deviations** of the mean in either direction

A measurement would be an extreme outlier if it fell more than 3 s.d. above or below the mean.

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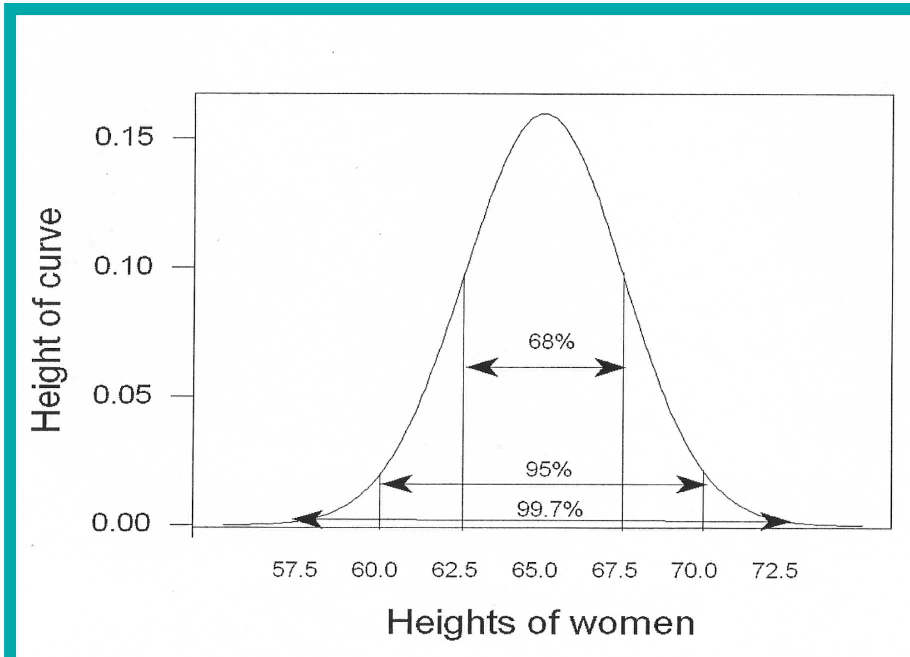
Heights of Adult Women

Since adult women in U.S. have a mean height of 65 inches with a s.d. of 2.5 inches and heights are bell-shaped, approximately …



- **68%** of adult women are between 62.5 and 67.5 inches,
- **95%** of adult women are between 60 and 70 inches,
- **99.7%** of adult women are between 57.5 and 72.5 inches.

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For Those Who Like Formulas

Notation for a Population

The lowercase Greek letter "mu" = μ represents the **population mean.**

The lowercase Greek letter "sigma" = σ represents the **population standard deviation.**

Therefore, the **population variance** is represented by σ^2.

A **normal distribution** with a mean of μ and variance of σ^2 is denoted by $N(\mu, \sigma^2)$.

For example, the **standard normal distribution** is denoted by $N(0, 1)$.

Standardized Score *z* for an Observed Value *x*

$$z = \frac{x - \mu}{\sigma}$$

Observed Value *x* for a Standardized Score *z*

$$x = \mu + z\sigma$$

Empirical Rule

If a population of values is $N(\mu, \sigma^2)$, then approximately:

68% of values fall within the interval $\mu \pm \sigma$

95% of values fall within the interval $\mu \pm 2\sigma$

99.7% of values fall within the interval $\mu \pm 3\sigma$

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