



Chapter 20

Estimating Proportions with Confidence

Thought Question 1:



95% confidence interval for the proportion of British couples in which the wife is taller than the husband: 0.02 to 0.08 *or* 2% to 8%.

What do you think it means to say that the interval from 0.02 to 0.08 represents a 95% confidence interval for the proportion of couples in which the wife is taller than the husband?

Thought Question 2:



Do you think a **99% confidence interval** for the proportion described in Question 1 would be **wider or narrower than the 95% interval** given?
Explain.

Thought Question 3:



In a Yankelovich Partners poll of **1000 adults** (*USA Today*, 20 April 1998), 45% reported that they believed in “faith healing.”

Based on this survey, a “**95% confidence interval**” for the proportion in the population who believe is about **42% to 48%**.

If this poll had been based on **5000 adults** instead, do you think the “**95% confidence interval**” would be **wider or narrower** than the interval given?

Explain.

Thought Question 4:



How do you think the concept of *margin of error*, explained in Chapter 4, relates to confidence intervals for proportions?

As a concrete example, can you determine the margin of error for the situation in Question 1 from the information given? In Question 3?

20.1 Confidence Intervals



Confidence Interval: *an interval of values computed from sample data that is almost sure to cover the true population number.*

- Most **common level of confidence used is 95%**. So willing to take a 5% risk that the interval does not actually cover the true value.
- We **never know for sure** whether any one given confidence interval covers the truth. However, ...
- **In long run**, 95% of all confidence intervals tagged with 95% confidence will be correct and 5% of them will be wrong.

20.2 Three Examples of Confidence Intervals from Media

the



Most polls report a margin of error along with the proportion of the sample that had each opinion.

Formula for a **95% confidence interval**:

sample proportion \pm margin of error

Example 1: A Public Opinion Poll

Poll reported in *The Sacramento Bee* (Nov 19, 2003, p. A20)
54% of respondents agreed that gay and lesbian couples
could be good parents. Report also provided ...

*Source: Pew Research Center for the People and the Press survey of
1,515 U.S. adults, Oct 15-19; margin of error 3 percentage points.*

What proportion of entire adult population at that time would
agree that gay and lesbian couples could be good parents?

A **95% confidence interval** for that population proportion:

sample proportion \pm margin of error

59% \pm 5%

54% to 64%

Interval resides above 50% \Rightarrow a majority of adult Americans
in 2003 believed such couples can be good parents.

Example 2: Number of AIDS Cases in U.S.

Story reported in the *Davis (CA) Enterprise* (14 December 1993, p. A5)

For the first time, survey data is now available on a randomly chosen cross section of Americans. Conducted by the National Center for Health Statistics, it concludes that 550,000 Americans are actually infected.

Some estimates had been as high as 10 million people, but the CDC had estimated the number at about 1 million.

Could the results of this survey rule out the fact that the number of people infected may be as high as 10 million?

*Dr. Geraldine McQuillan, who presented the analysis, said the **statistical margin of error** in the survey suggests that the **true figure** is probably **between 300,000 and just over 1 million people**. “The real number may be a little under or a bit over the CDC estimate” [of 1 million], she said, **“but it is not 10 million.”***

Example 3: The Debate Over Passive Smoking

U.S. EPA says there is a 90% probability that the risk of lung cancer for passive smokers is somewhere between 4% and 35% higher than for those who aren't exposed to environmental smoke. To statisticians, this calculation is called the "90% confidence interval."

And that, say tobacco-company statisticians, is the rub. "Ninety-nine percent of all epidemiological studies use a 95% confidence interval," says Gio B. Gori, director of the Health Policy Center in Bethesda, Md.

Problem: EPA used 90% because the amount of data available at the time did not allow an extremely accurate estimate of true change in risk of lung cancer for passive smokers. The 95% confidence interval actually went below zero percent, indicating it's possible passive smoke *reduces* risk.

The EPA believes it is inconceivable that breathing in smoke containing cancer-causing substances could be healthy and any hint in the report that it might be would be meaningless and confusing. (p. B-4)

Source: Wall Street Journal (July 28, 1993, pp. B-1, B-4)

20.3 Constructing a Confidence Interval for a Proportion

Recall Rule for Sample Proportions:

If numerous samples or repetitions of same size are taken, the frequency curve made from proportions from various samples will be **approximately bell-shaped**.

Mean will be true proportion from the population.

Standard deviation will be:

$$\sqrt{\frac{(\text{true proportion})(1 - \text{true proportion})}{\text{sample size}}}$$

In 95% of all samples, the sample proportion will fall within 2 standard deviations of the mean, which is the true proportion.

A Confidence Interval for a Proportion

In 95% of all samples, the true proportion will fall within 2 standard deviations of the sample proportion.

Problem: Standard deviation uses the unknown “true proportion”.

Solution: Substitute the sample proportion for the true proportion in the formula for the standard deviation.

A 95% confidence interval for a population proportion:

sample proportion \pm 2(S.D.)

Where S.D. = $\sqrt{\frac{(\text{true proportion})(1 - \text{true proportion})}{\text{sample size}}}$

A technical note: To be exact, we would use 1.96(S.D.) instead of 2(S.D.). However, rounding 1.96 off to 2.0 will not make much difference.

Example 4: Wife Taller than the Husband?

In a random sample of 200 British couples, the wife was taller than the husband in only 10 couples.

- **sample proportion** = $10/200 = 0.05$ or 5%
- **standard deviation** = $\sqrt{\frac{(0.05)(1 - 0.05)}{200}} = 0.015$
- **confidence interval** = $.05 \pm 2(0.015) = .05 \pm .03$
or **0.02 to 0.08**

Interpretation: We are 95% confident that of all British couples, between .02 (2%) and .08 (8%) are such that the wife is taller than her husband.

Example 5: Experiment in ESP

Experiment: Subject tried to guess which of four videos the “sender” was watching in another room. Of the 165 cases, 61 resulted in successful guesses.

- **sample proportion = $61/165 = 0.37$ or 37%**
- **standard deviation = $\sqrt{\frac{(0.37)(1 - 0.37)}{165}} = 0.038$**
- **confidence interval = $.37 \pm 2(0.038) = .37 \pm .08$
or 0.29 to 0.45**

Interpretation: We are 95% confident that the probability of a successful guess in this situation is between .29 (29%) and .45 (45%). Notice this interval lies entirely above the 25% value expected by chance.

Example 6: Quit Smoking with the Patch

Study: Of 120 volunteers randomly assigned to use a nicotine patch, 55 had quit smoking after 8 weeks.

- **sample proportion = $55/120 = 0.46$ or 46%**
- **standard deviation = $\sqrt{\frac{(0.46)(1 - 0.46)}{120}} = 0.045$**
- **confidence interval = $.46 \pm 2(0.045) = .46 \pm .09$
or **0.37 to 0.55****

Interpretation: We are 95% confident that between 37% and 55% of smokers treated in this way would quit smoking after 8 weeks. The placebo group confidence interval is 13% to 27%, which does not overlap with the nicotine patch interval.

Other Levels of Confidence



- **68% confidence interval:**
sample proportion ± 1 (S.D.)
- **99.7% confidence interval:**
sample proportion ± 3 (S.D.)
- **90% confidence interval:**
sample proportion ± 1.645 (S.D.)
- **99% confidence interval:**
sample proportion ± 2.576 (S.D.)

How the Margin of Error was Derived



Two formulas for a 95% confidence interval:

- sample proportion $\pm 1/\sqrt{n}$ (from conservative m.e. in Chapter 4)
- sample proportion $\pm 2(\text{S.D.})$

Two formulas are equivalent when the proportion used in the formula for S.D. is 0.50.

Then $2(\text{S.D.})$ is simply $1/\sqrt{n}$, which is our conservative formula for margin of error – called *conservative* because the true margin of error is actually likely to be smaller.

Case Study 20.1: A Winning Confidence Interval Loses in Court



- Sears company erroneously collected and paid city sales taxes for sales made to individuals outside the city limits.
- Sears took a random sample of sales slips for the period in question to estimate the proportion of all such sales.
- 95% confidence interval for true proportion of all sales made to out-of-city customers: $.367 \pm .03$, or .337 to .397.
- To estimate amount of tax owed, multiplied percentage by total tax they had paid of \$76,975. Result = \$28,250 with **95% confidence interval from \$25,940 to \$30,559.**

Source: Gastwirth (1988, p. 495)

Case Study 20.1: A Winning Confidence Interval Loses in Court



- Judge did not accept use of sampling and required Sears to examine all sales records.
- Total they were owed about \$27,586. Sampling method Sears had used provided a fairly accurate estimate of the amount they were owed.
- Took Sears 300 person-hours to conduct the sample and 3384 hours to do the full audit.
- In fairness, the judge in this case was simply following the law; the sales tax return required a sale-by-sale computation.

A well designed sampling audit may yield a more accurate estimate than a less carefully carried out complete audit.

Source: Gastwirth (1988, p. 495)

For Those Who Like Formulas



Notation for Population and Sample Proportions

Sample size = n

Population proportion = p

Sample proportion = \hat{p}

Notation for the Multiplier for a Confidence Interval

For reasons that will become clear in later chapters, we specify the *level* of confidence for a confidence interval as $(1 - \alpha$ (read “alpha”)) 100%. For example, for a 95% confidence interval, $\alpha = .05$. Let $z_{\alpha/2}$ = standardized normal score with area $\alpha/2$ above it. Then the area between $z_{\alpha/2}$ and $-z_{\alpha/2}$ is $1 - \alpha$. For example, when $\alpha = .05$, as for a 95% confidence interval, $z_{\alpha/2} = 1.96$, or about 2.

Formula for a $(1 - \alpha)$ 100% Confidence Interval for a Proportion

$$\hat{p} \pm z_{\alpha/2} \sqrt{\frac{\hat{p}(1 - \hat{p})}{n}}$$

Common Values of $z_{\alpha/2}$

1.0 for a 68% confidence interval

1.96 or 2.0 for a 95% confidence interval

1.645 for a 90% confidence interval

2.576 for a 99% confidence interval

3.0 for a 99.7% confidence interval