Statistical challenges in neural data analysis

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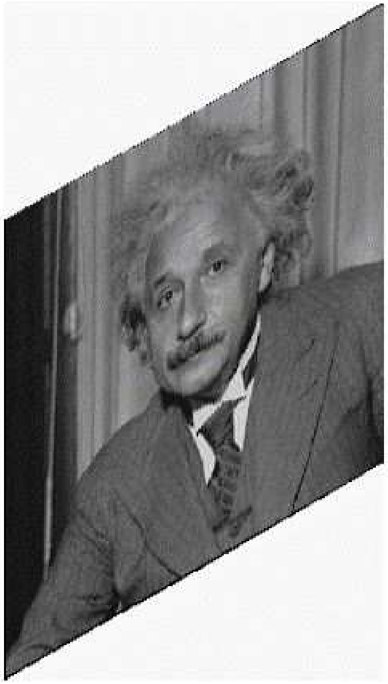
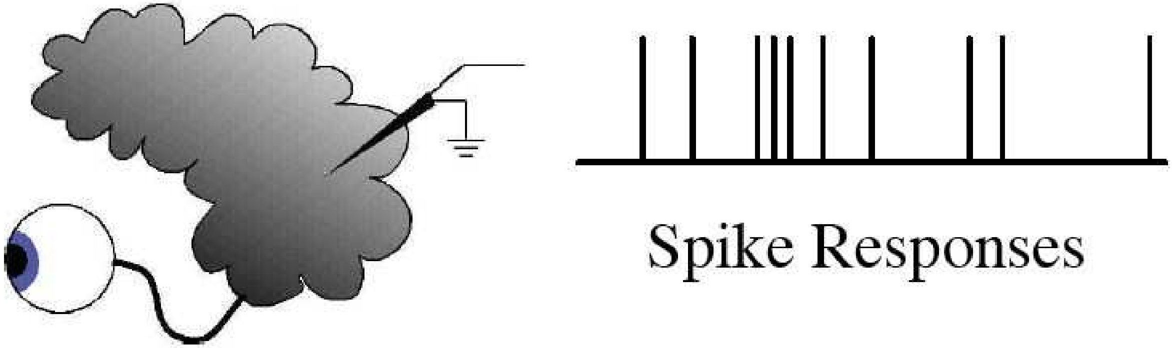
The neural code





Basic goal: infer input-output relationship between

- External observables x (sensory stimuli, motor responses...)
- Neural variables y (spike trains, population activity...)

Several levels of neural data analysis

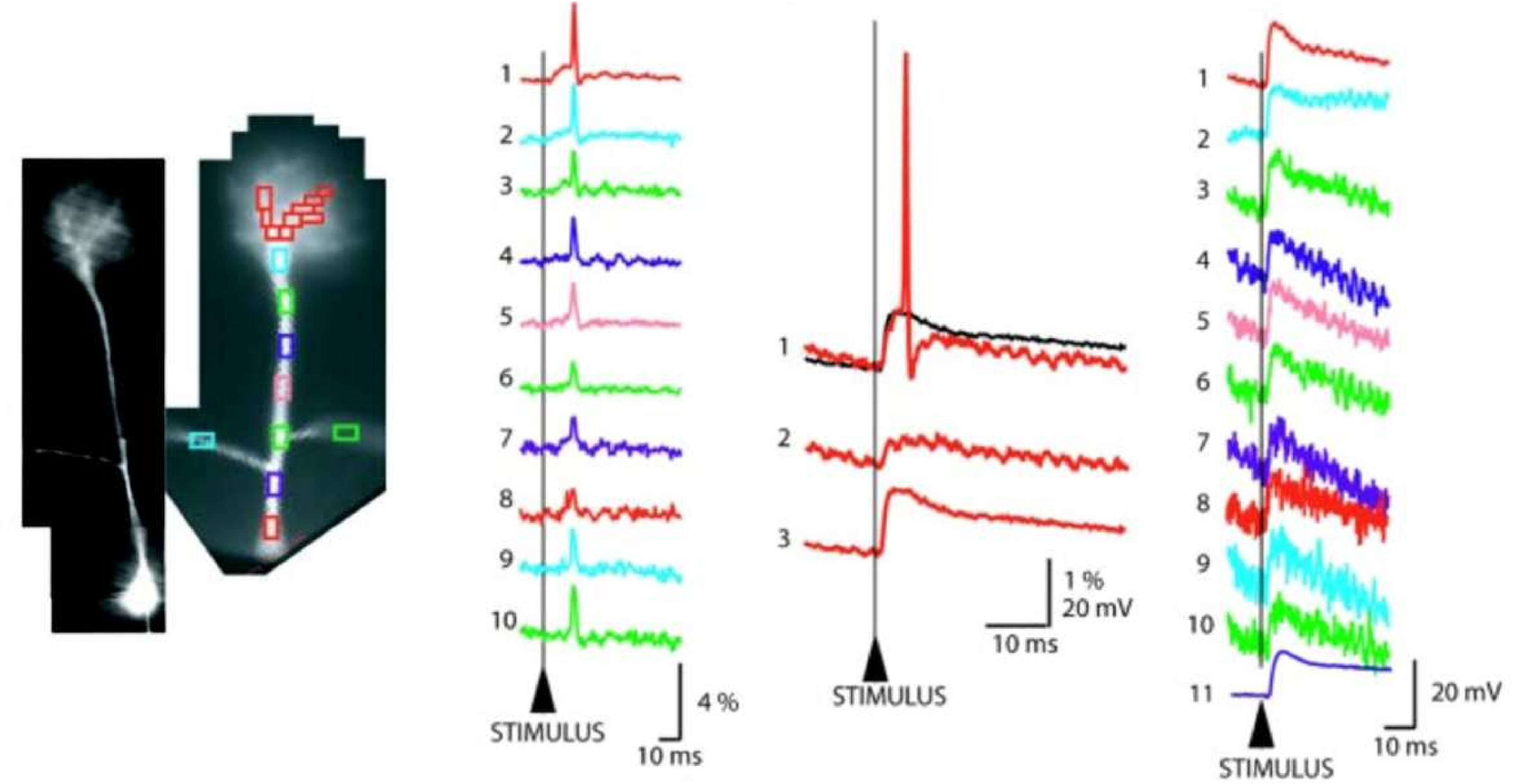
- "Subcellular" level: measurements of intracellular voltage or ionic concentrations (intracellular "patch" electrodes, two-photon imaging, molecular tagging)
- "Circuit" level: electrical activity of single neurons or small groups of isolated neurons (multi-electrode recordings, calcium-sensitive microscopy)
- "Systems" level: blood flow or other indirect measurements of electrical activity in coarsely-defined brain areas (fMRI, EEG, MEG...)

Three challenges

1. Reconstructing the full spatiotemporal voltage on a dendritic tree given noisy, intermittently-sampled subcellular measurements
2. Decoding behaviorally-relevant information from multiple spike trains
3. Inferring connectivity from large populations of noisily-observed spike trains

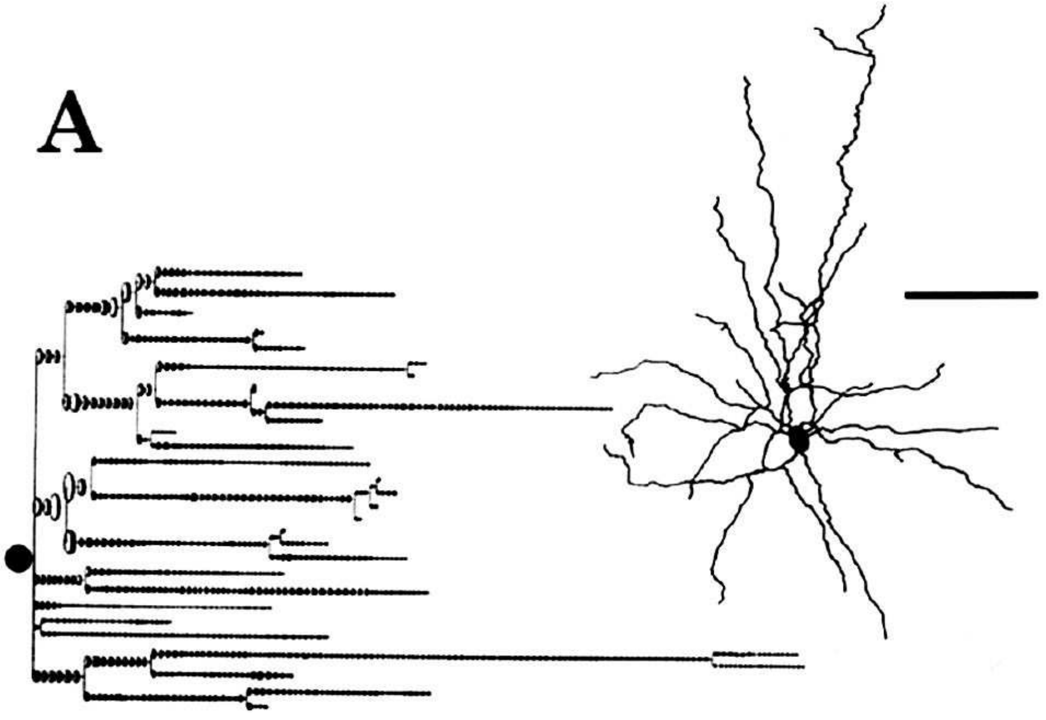
The filtering problem

Spatiotemporal imaging data opens an exciting window on the computations performed by single neurons, but we have to deal with noise and intermittent observations.



(Djurisic et al., 2004)

Basic paradigm: compartmental models



- write neuronal dynamics in terms of equivalent nonlinear, time-varying RC circuits
- leads to a coupled system of stochastic differential equations

Inference of spatiotemporal neuronal state given noisy observations

State-space approach: q_t = state of neuron at time t.

We want $p(q_t|Y_{1:t}) \propto p(q_t, Y_{1:t})$. Markov assumption:

$$p(Q,Y) = p(Q)p(Y|Q) = p(q_1)\left(\prod_{t=2}^{T} p(q_t|q_{t-1})\right)\left(\prod_{t=1}^{T} p(y_t|q_t)\right)$$

To compute $p(q_t, Y_{1:t})$, just recurse

$$p(q_t, Y_{1:t}) = p(y_t|q_t)\int_{q_{t-1}} p(q_t|q_{t-1})p(q_{t-1}, Y_{1:t-1})dq_{t-1}.$$

Linear-Gaussian case: requires $O(\dim(q)^3 T)$ time; in principle, just matrix algebra (Kalman filter). Approximate solutions in more general case via sequential Monte Carlo (Huys and Paninski, 2009).

Major challenge: $\dim(q)$ can be $\approx 10^4$ or greater.

Low-rank approximations

Key fact: current experimental methods provide just a few low-SNR observations per time step.

Basic idea: if dynamics are approximately linear and time-invariant, we can approximate Kalman covariance $C_t = cov(q_t|Y_{1:t})$ as a perturbation of the marginal covariance $C_0 + U_t D_t U_t^T$, with $C_0 = \lim_{t\to\infty} cov(q_t)$.

C_0 is the solution to a Lyapunov equation. It turns out that we can solve linear equations involving C_0 in $O(\dim(q))$ time via Gaussian belief propagation, using the fact that the dendrite is a tree.

The necessary recursions — i.e., updating U_t, D_t and the Kalman mean $E(q_t|Y_{1:t})$ — involve linear manipulations of C_0, using

$$\begin{aligned} C_t &= [(AC_{t-1}A^T + Q)^{-1} + B_t]^{-1} \\ C_0 + U_t D_t U_t^T &= \left([A(C_0 + U_{t-1}D_{t-1}U_{t-1}^T)A^T + Q]^{-1} + B_t\right)^{-1}, \end{aligned}$$

and can be done in $O(\dim(q))$ time (Paninski, 2010).

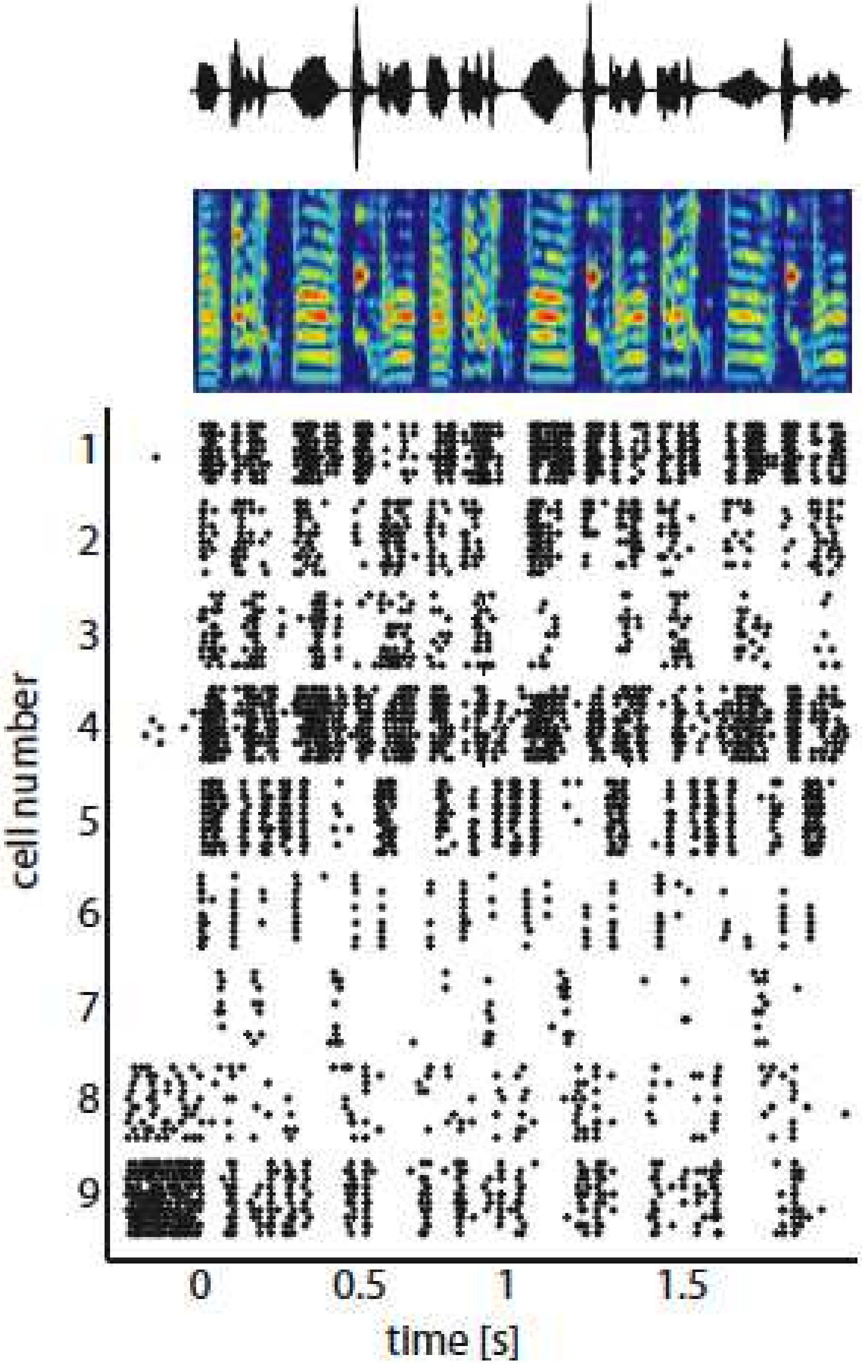
Example: inferring voltage from subsampled observations

(Loading low-rank-speckle.mp4)

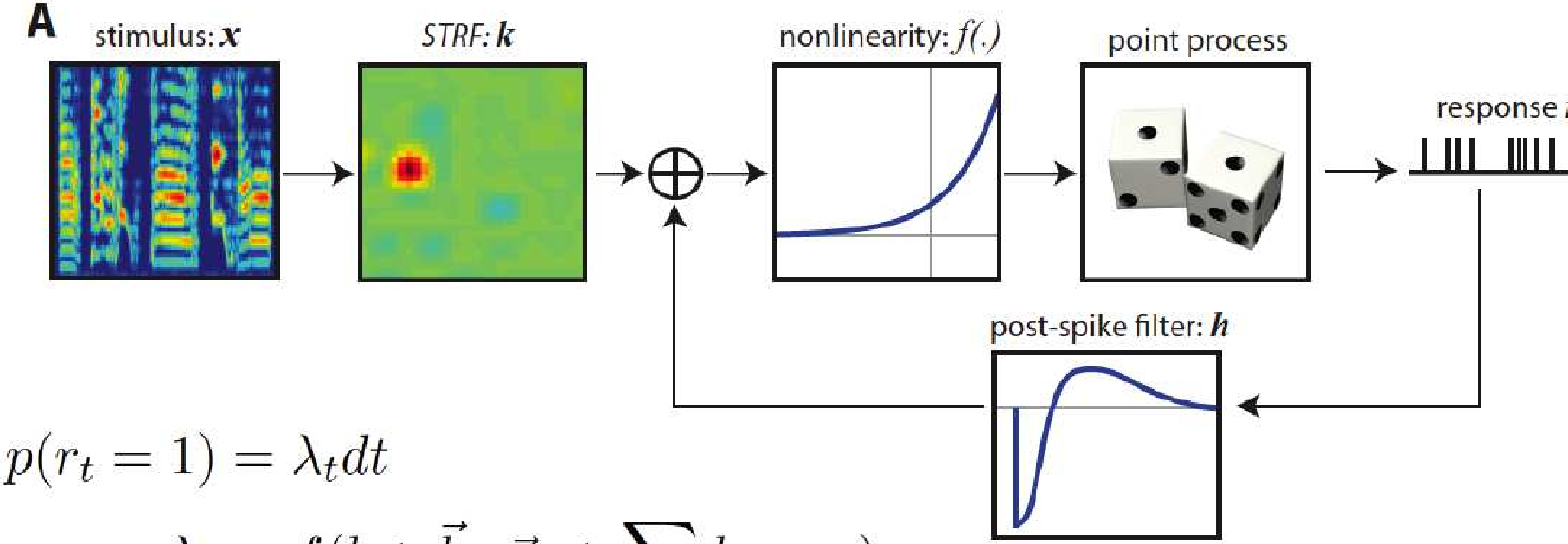
Applications

- Optimal experimental design: which parts of the neuron should we image? (Submodular optimization; Krause and Guestrin, '07)
- Estimation of biophysical parameters (e.g., membrane channel densities, axial resistance, etc.): reduces to a simple nonnegative regression problem once $V(x, t)$ is known (Huys et al., 2006)
- Detecting location and weights of synaptic input

Part 2: optimal decoding of spike train data



Semiparametric GLM



$$p(r_t = 1) = \lambda_t dt$$

$$\lambda_t = f(b + \vec{k} \cdot \vec{x}_t + \sum_j h_j r_{t-j})$$

Parameters $(\vec{k}, h)$ estimated by L_1-penalized maximum likelihood (concave); f estimated by log-spline (Calabrese, Woolley et al. 2009). Currently the best predictive model of these spike trains.

MAP stimulus decoding

It is reasonable to estimate the song X that led to a response R via the MAP

$$\hat{X} = \arg\max_X p(X|R).$$

(Note that X is very high-dimensional!) For this model, we have:

$$\begin{aligned} \log p(X|R) &= \log p(X) + \log p(R|X) + const. \\ &= \log p(X) + \sum_t \log p(r_t|X, R_{\ldots,t-1}) + const. \end{aligned}$$

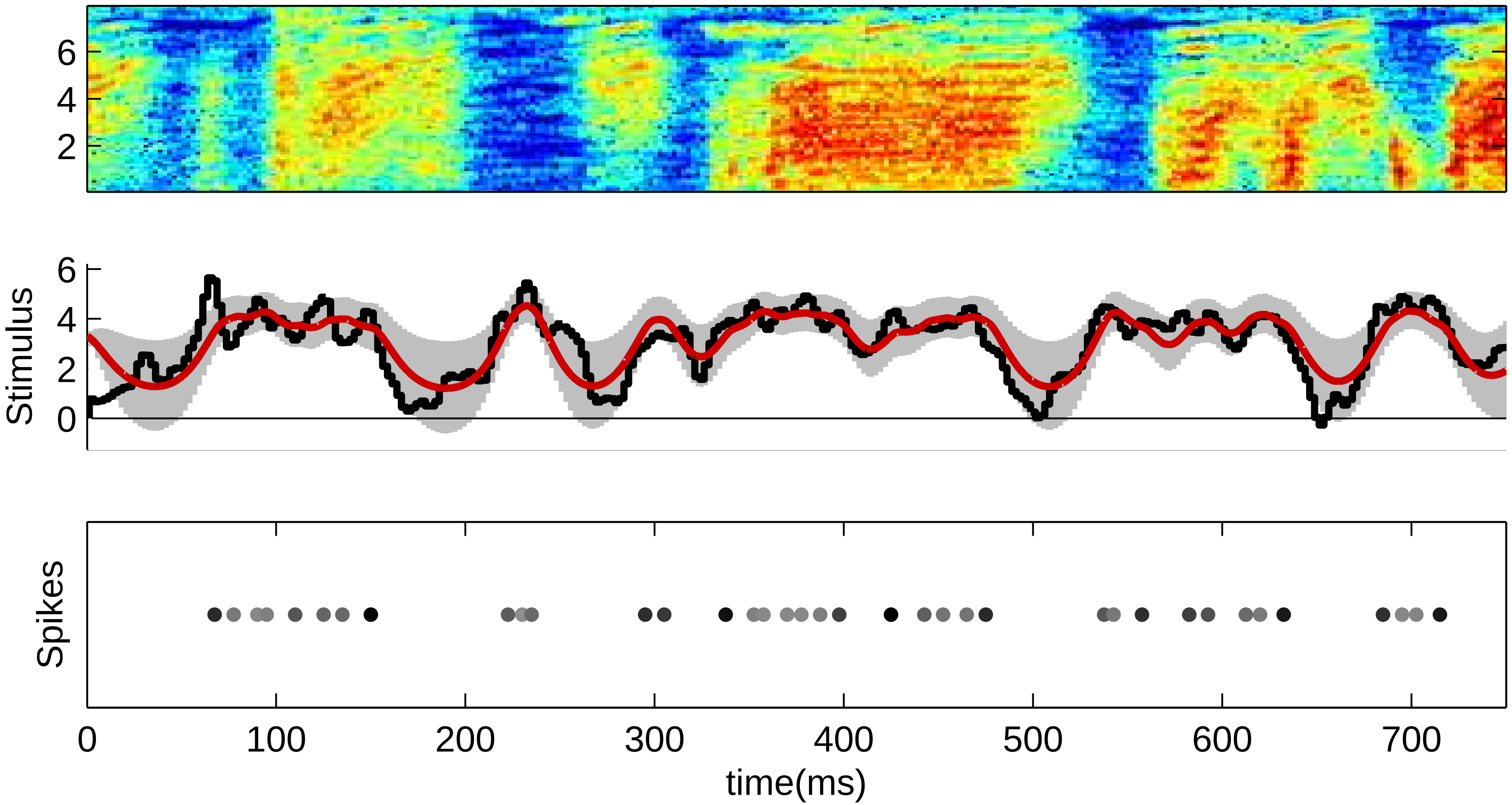
Two basic observations:

- If $\log p(X)$ is concave, then so is $\log p(X|R)$, since each $\log p(r_t|X, Y_{\ldots,t-1})$ is.
- Hessian H of $\log p(R|X)$ w.r.t. X is banded: each $p(r_t|X, R_{\ldots,t-1})$ depends on X locally in time, and therefore contributes a banded term.

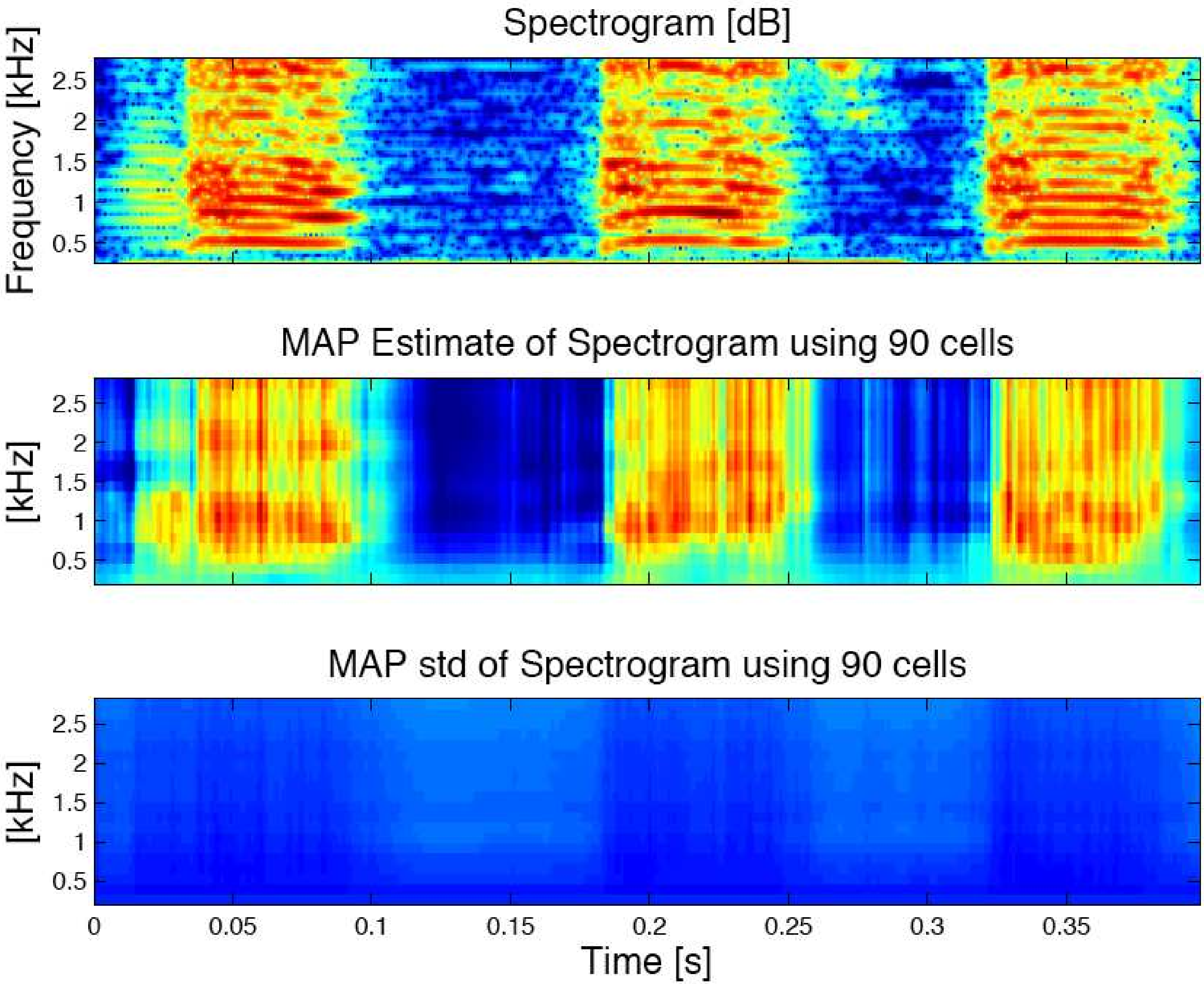
Newton's method iteratively solves $HX_{dir} = \nabla$. Solving banded systems requires $O(T)$ time, so computing MAP requires $O(T)$ time if log-prior is concave with a banded Hessian.

— similar speedups available in constrained cases (Paninski et al., 2010), and in MCMC setting (e.g., fast hybrid Monte Carlo methods (Ahmadian et al., 2010b)).

Application: fast optimal decoding



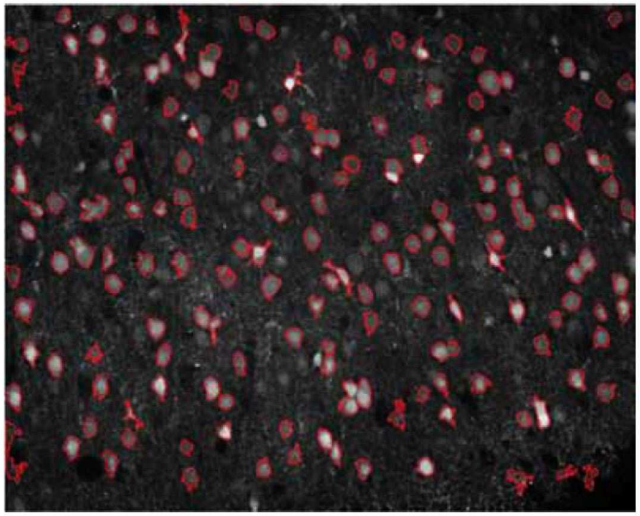
Decoding a full song



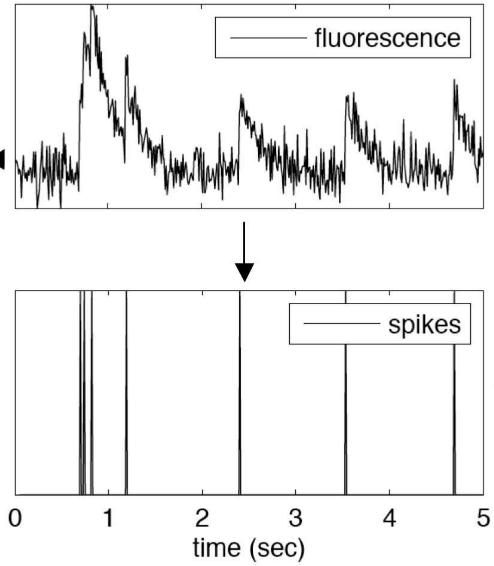
(Ramirez et al., 2010)

Part 3: circuit inference

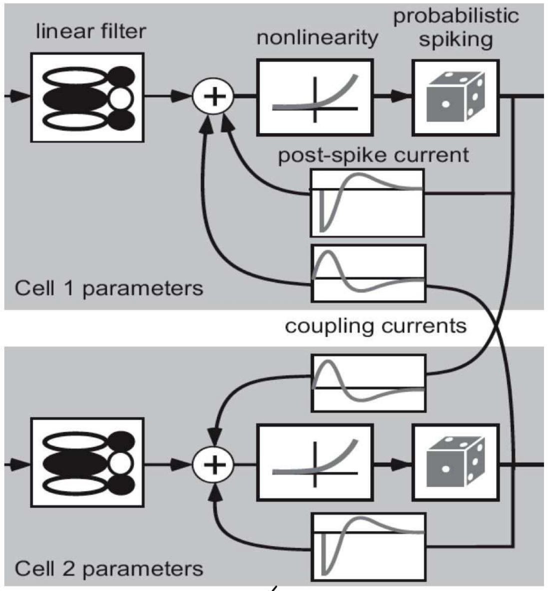
Record large-scale calcium movie



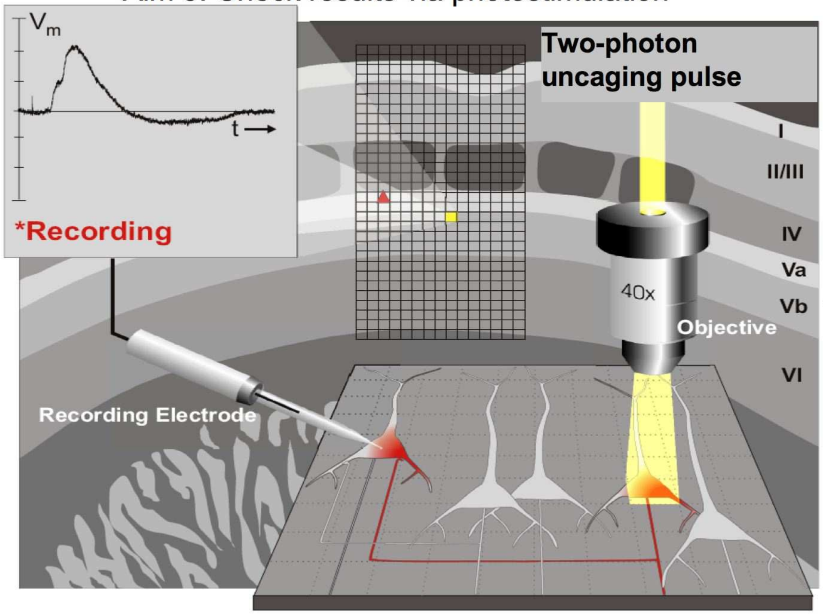
Aim 1: Extract spike times



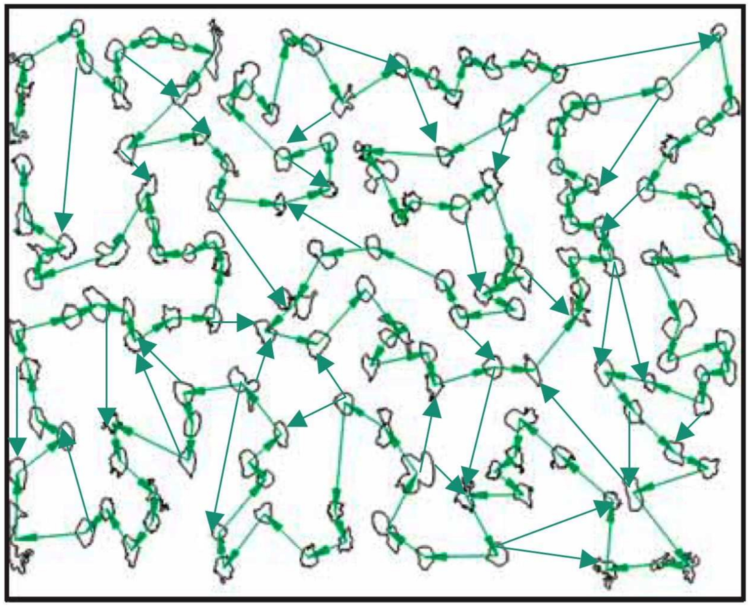
Aim 2: Estimate network model



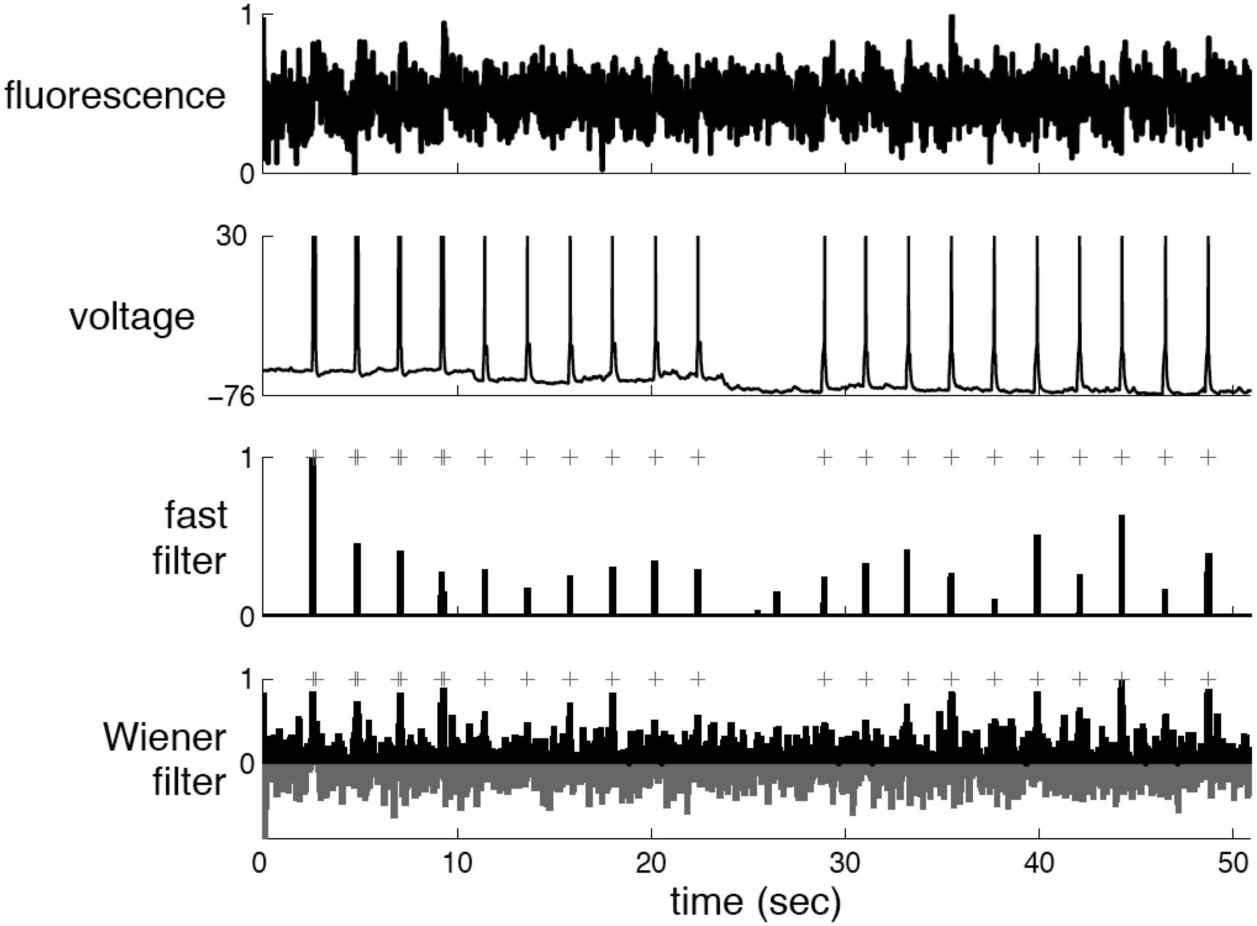
Aim 3: Check results via photostimulation



Inferred network model



Challenge: slow, noisy calcium data



First-order model:

$$C_{t+dt} = C_t - dt C_t/\tau + r_t;\ r_t > 0;\ y_t = C_t + \epsilon_t$$

— $\tau \approx 100$ ms; nonnegative deconvolution problem. Can be solved by $O(T)$ relaxed constrained interior-point optimization (Vogelstein et al., 2010) or sequential Monte Carlo (Vogelstein et al., 2009).

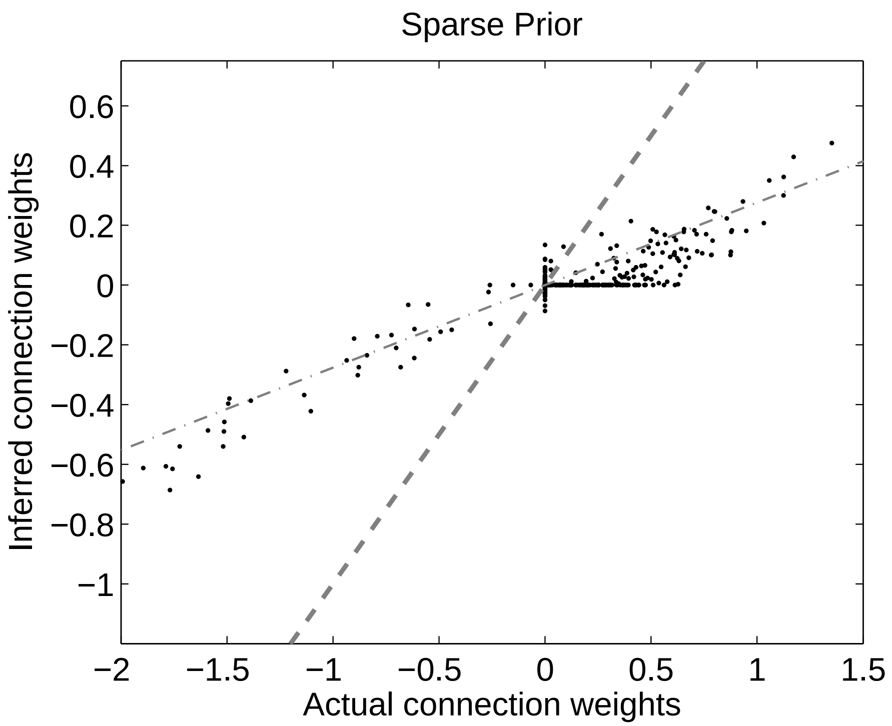
Monte Carlo EM approach

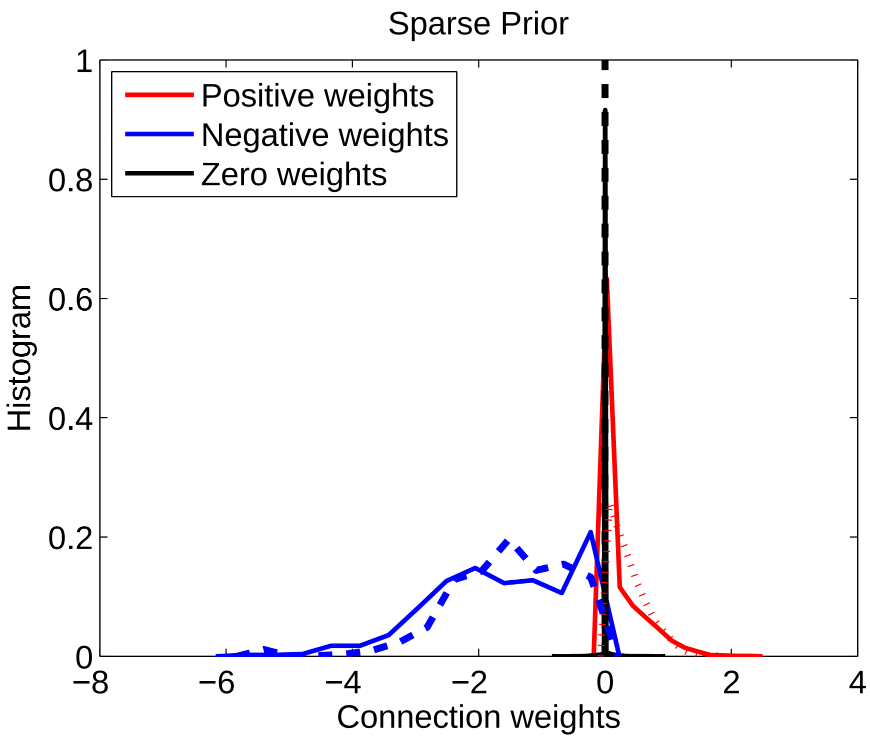
Given the spike times in the network, L_1-penalized likelihood optimization is easy. But we only have noisy calcium observations Y; true spike times are hidden variables. Thus an EM approach is natural.

- E step: sample spike train responses R from $p(R|Y, \theta)$
- M step: given sampled spike trains, perform L_1-penalized likelihood optimization to update parameters θ.

E step is hard part here. Use the fact that neurons interact fairly weakly; thus we need to sample from a collection of weakly-interacting Markov chains, via Metropolis-within-blockwise-Gibbs forward-backward methods (Neal et al., 2003).

Simulated circuit inference





— Connections are inferred with the correct sign in conductance-based integrate-and-fire networks with biologically plausible connectivity matrices (Mishchencko et al., 2009).

Good news: connections are inferred with the correct sign. But process is slow; current work focusing on improved sampling methods (exploiting hybrid forward-backward blockwise-Gibbs approach).

Optimal control of spike timing

Optimal experimental design and neural prosthetics applications require us to perturb the network at will. How can we make a neuron fire exactly when we want it to?

Assume bounded inputs; otherwise problem is trivial.

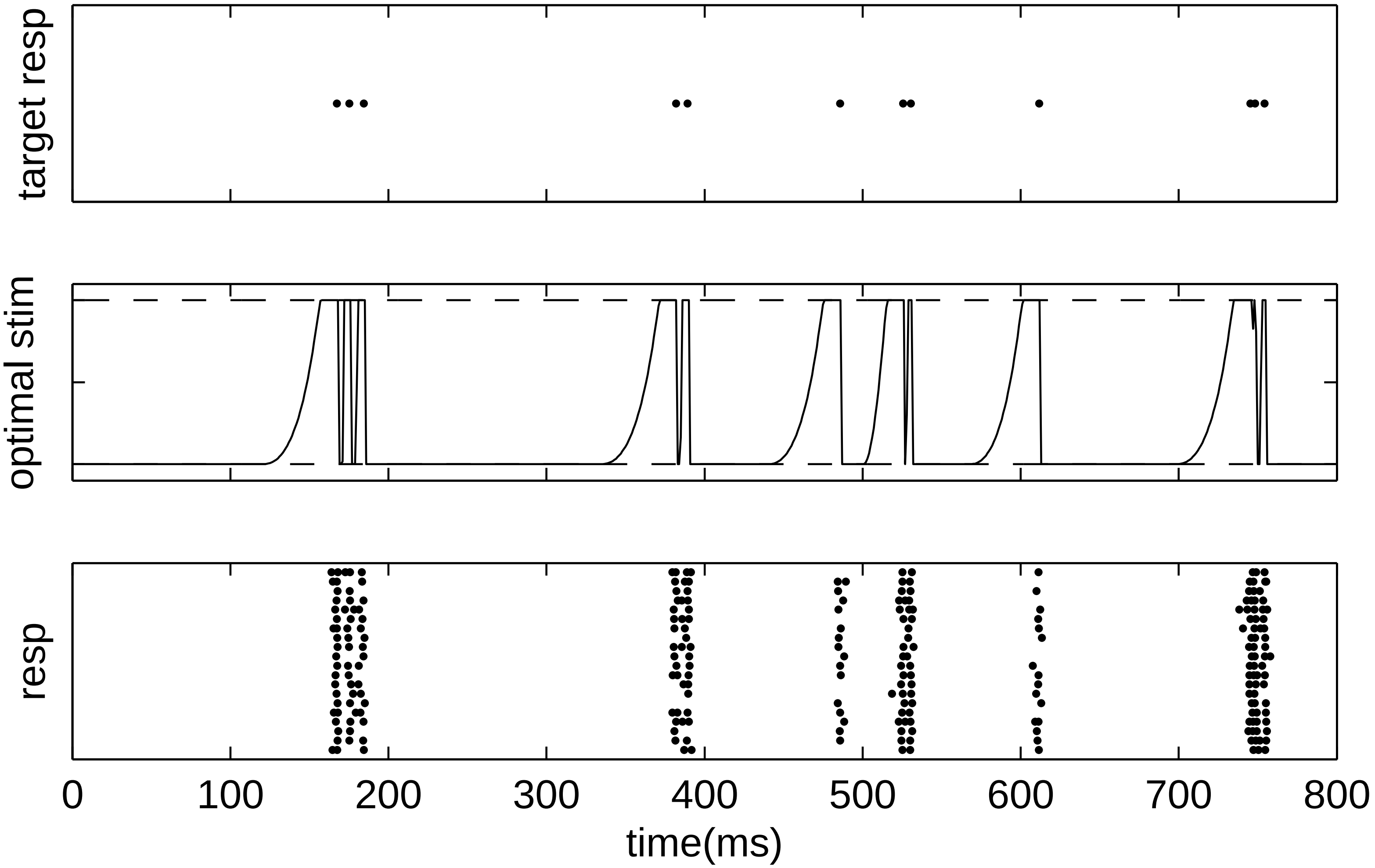
Start with a simple model:

$$\lambda_t \quad = \quad f(\vec{k} * I_t + h_t).$$

Now we can just optimize the likelihood of the desired spike train, as a function of the input I_t, with I_t bounded.

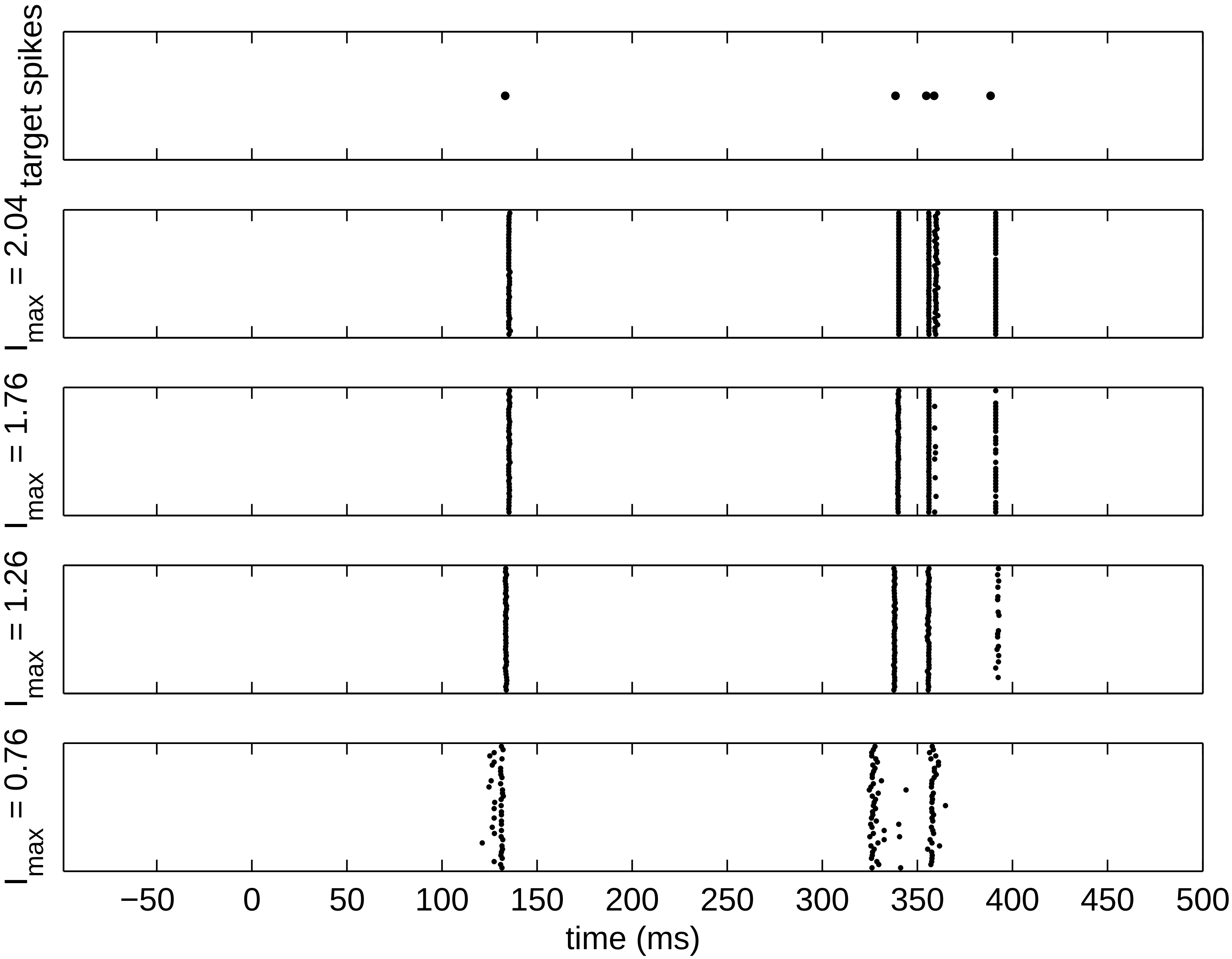
Concave objective function over convex set of possible inputs I_t + Hessian is banded $\implies O(T)$ optimization.

Optimal electrical control of spike timing



Extension to optical stimulation methods is straightforward
(Ahmadian et al., 2010a).

Example: intracellular control of spike timing



(Ahmadian et al., 2010a)

Conclusions

- GLM and state-space approaches provide flexible, powerful methods for answering key questions in neuroscience
- Close relationships between encoding, decoding, and experimental design (Paninski et al., 2007)
- Log-concavity, banded matrix methods make computations very tractable
- Experimental methods progressing rapidly; many new challenges and opportunities for breakthroughs based on statistical ideas

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