Statistical models for neural encoding, decoding, and optimal stimulus design

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Reinterpreting the STRF

Classic method for estimating spectrotemporal receptive field: fit the linear-Gaussian regression model

$$n_t = \vec{k} \cdot \vec{x}_t + \epsilon_t, \;\; \epsilon_t \sim \mathcal{N}(0, \sigma^2).$$

The STRF $\vec{k}$ weights the stimulus $\vec{x}_t$; ϵ_t models variability of response n_t.

Pros:

- analytical solution for optimal $\hat{k}$.
- easy to incorporate prior assumptions on $\vec{k}$ (e.g., smoothness); Bayesian smoothing methods built in to STRFPak (Theunissen et al., 2001).

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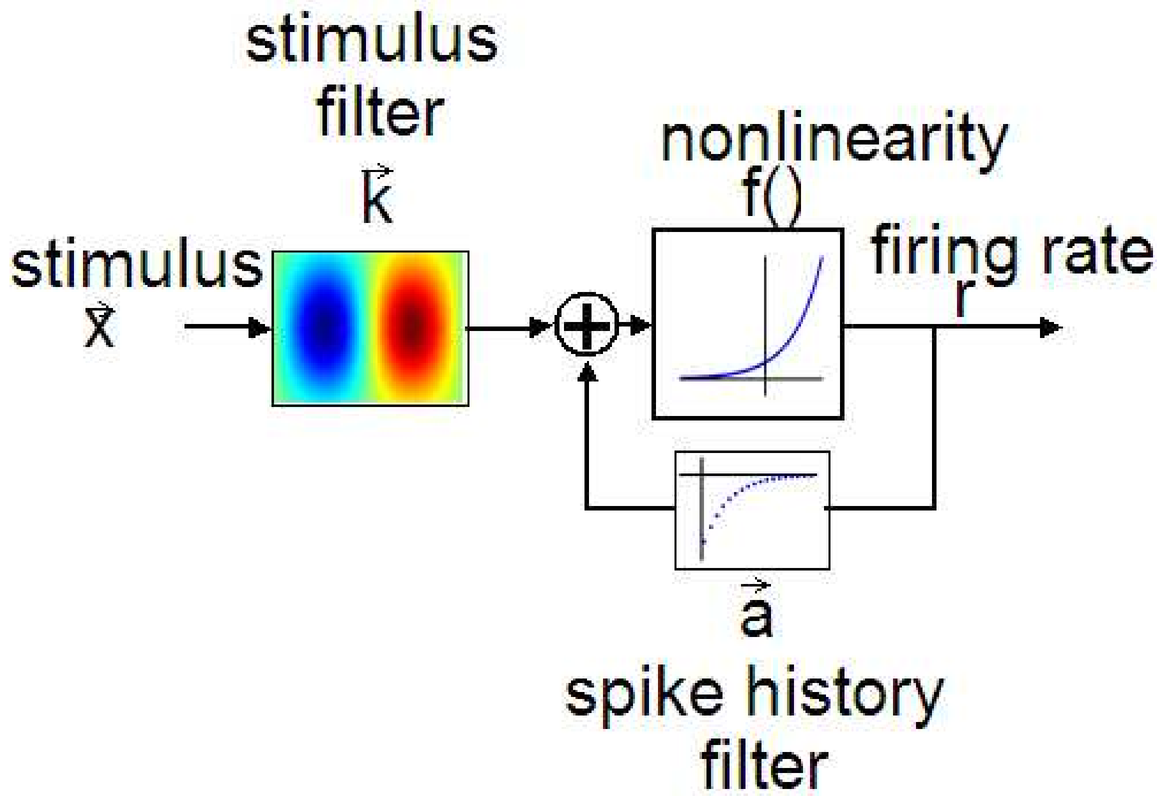
Cons:

- Gaussian model is not really accurate for spike trains.
- responses n_t can be negative.
- given stimulus $\vec{x}_t$, responses n_t are independent: no refractoriness, burstiness, firing-rate adaptation, etc.

Generalized linear model



$$p(n_t = 1) = \lambda_t dt$$

$$\lambda_t = f(\vec{k} \cdot \vec{x}_t + \sum_j a_j r_{t-j})$$

GLM likelihood

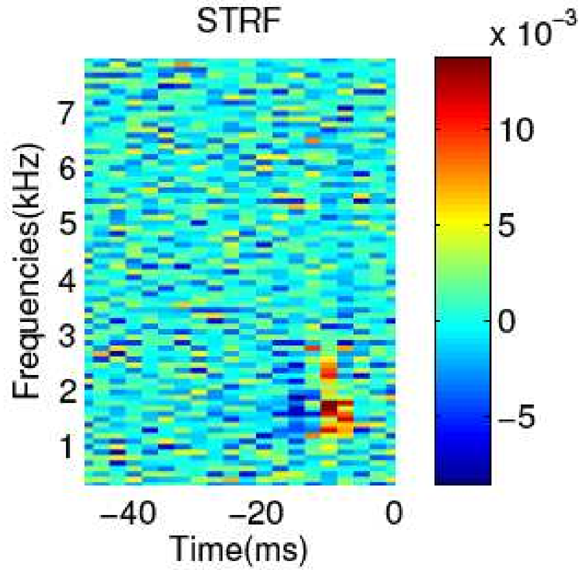
$$\lambda_t = f(\vec{k} \cdot \vec{x}_t + \sum_j a_j n_{t-j})$$

$$\log p(n_t|\vec{x}_t, \vec{\theta}) = -f(\vec{k}\cdot\vec{x}_t + \sum_j a_j n_{t-j}) + n_t \log f(\vec{k}\cdot\vec{x}_t + \sum_j a_j n_{t-j})$$

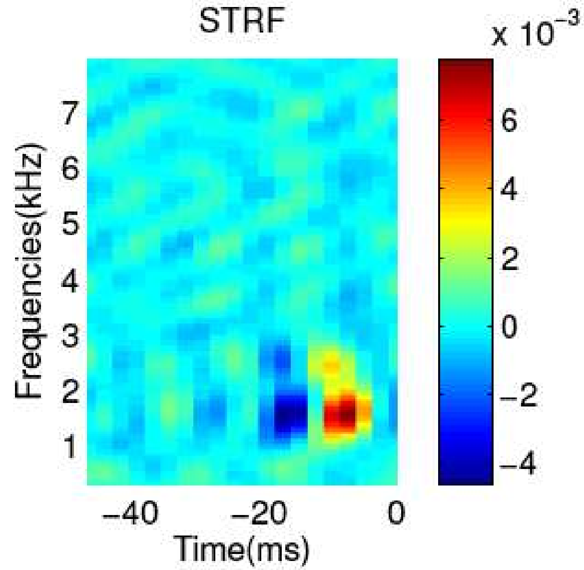
Key points:

- f convex and log-concave $\implies$ log-likelihood concave in $\vec{\theta}$. Easy to optimize, so estimating $\hat{\theta}$ is very tractable.
- Easy to include smoothing priors, as in STRFPak.
- Can also include nonlinear terms easily (Gill et al., 2006; Ahrens et al., 2008)

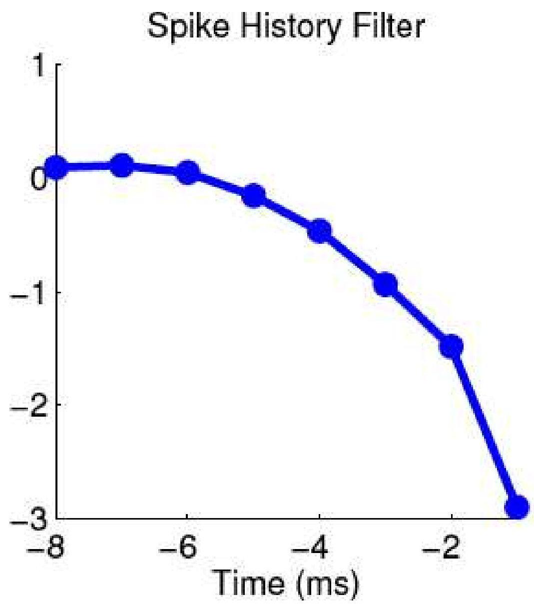
Estimated parameters



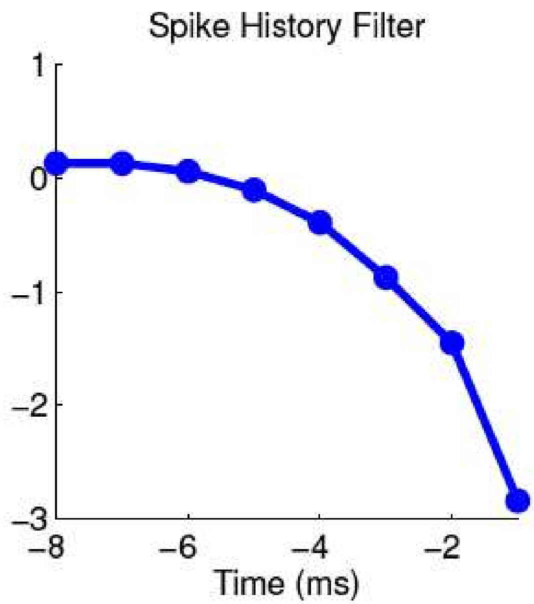
(a)



(b)

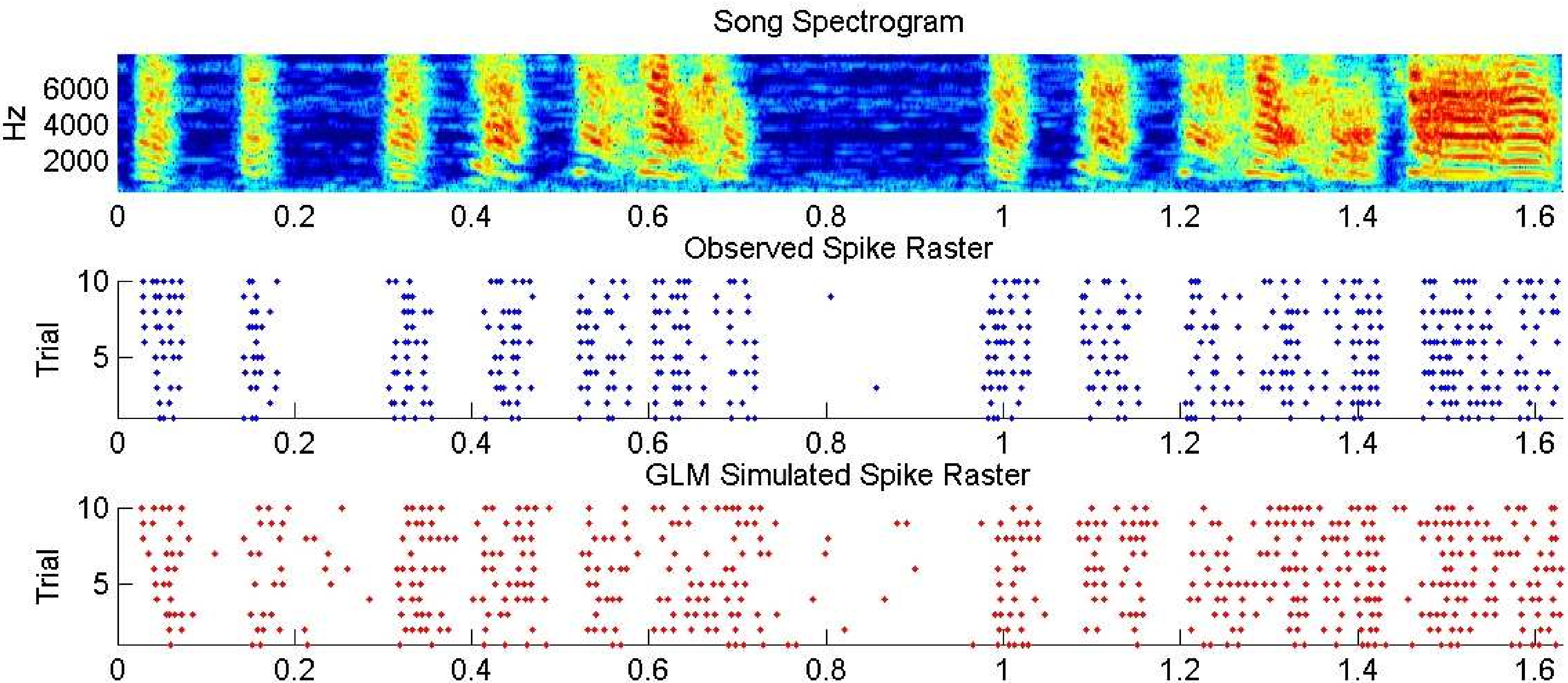


(c)



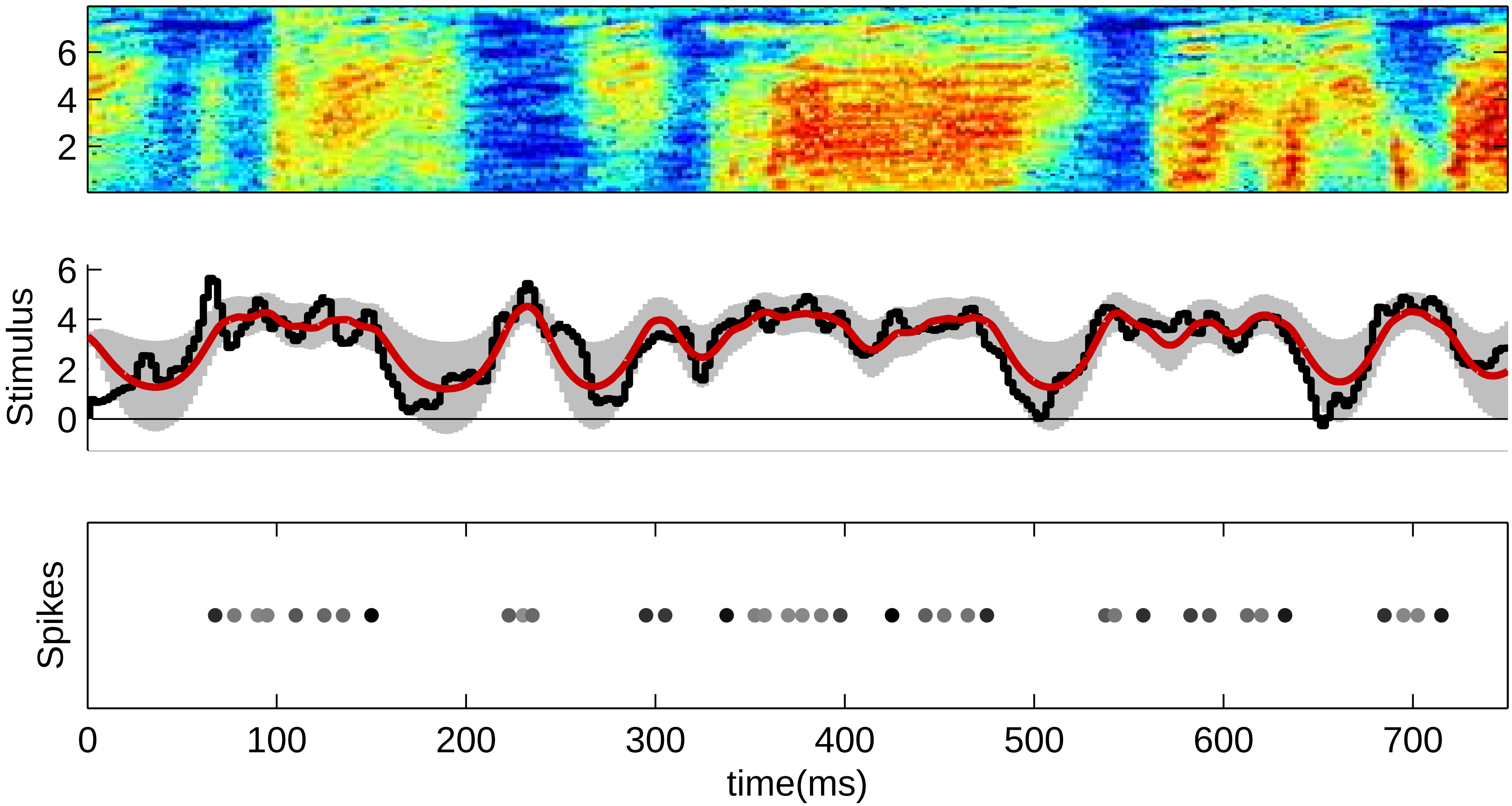
(d)

Model performance



Fast optimal decoding

Maximize $\log p(n|\vec{x}, \vec{\theta})$ with respect to $\vec{x}$. Concave optimization; only $O(T)$ time (Ahmadian et al., 2008b).



Optimal stimulus design

Idea: we have full control over the stimuli we present. Can we choose stimuli $\vec{x}_t$ to maximize the informativeness of each trial?

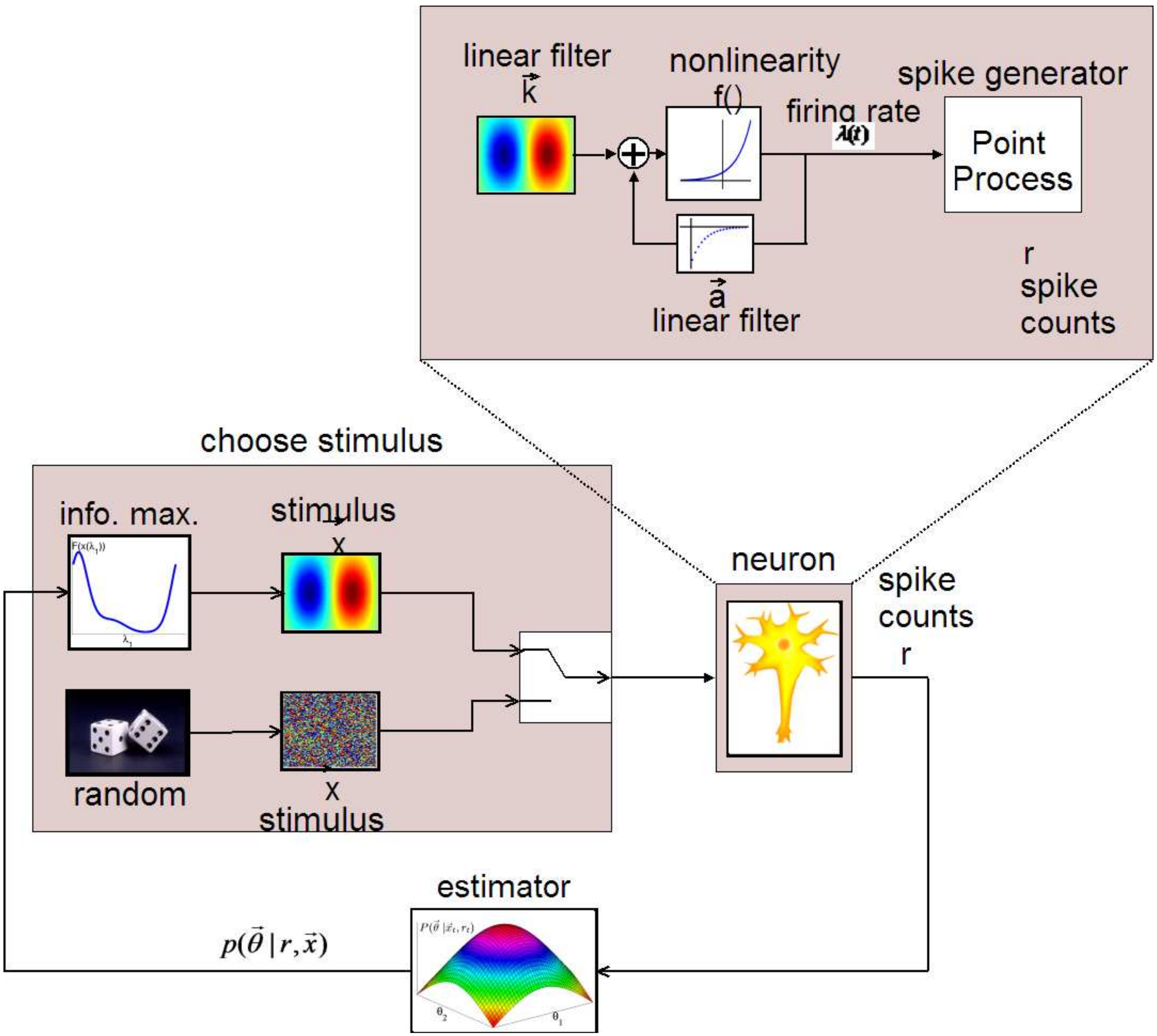
— More quantitatively, optimize $I(n_t; \theta | \vec{x}_t)$ with respect to $\vec{x}_t$.

Maximizing $I(n_t; \theta; \vec{x}_t) \implies$ minimizing uncertainty about θ.

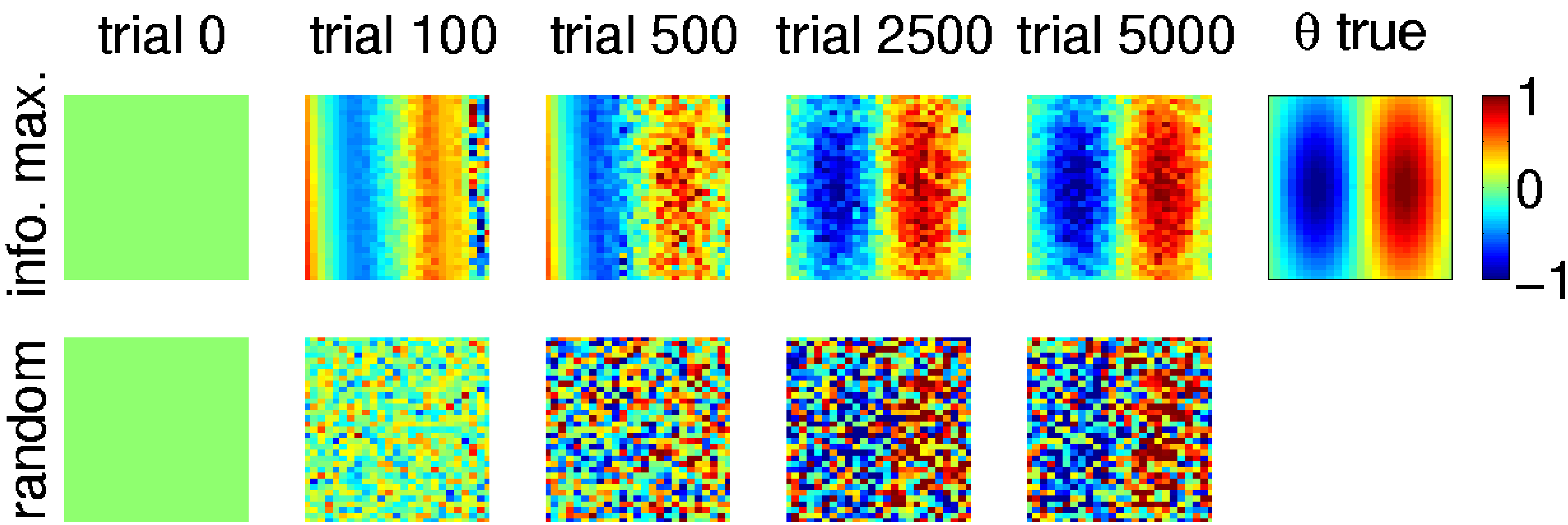
In general, very hard to do: high-d integration over θ to compute $I(n_t; \theta | \vec{x}_t)$, high-d optimization to select best $\vec{x}_t$.

GLM setting makes this surprisingly tractable
(Lewi et al., 2009).

Infomax vs. randomly-chosen stimuli

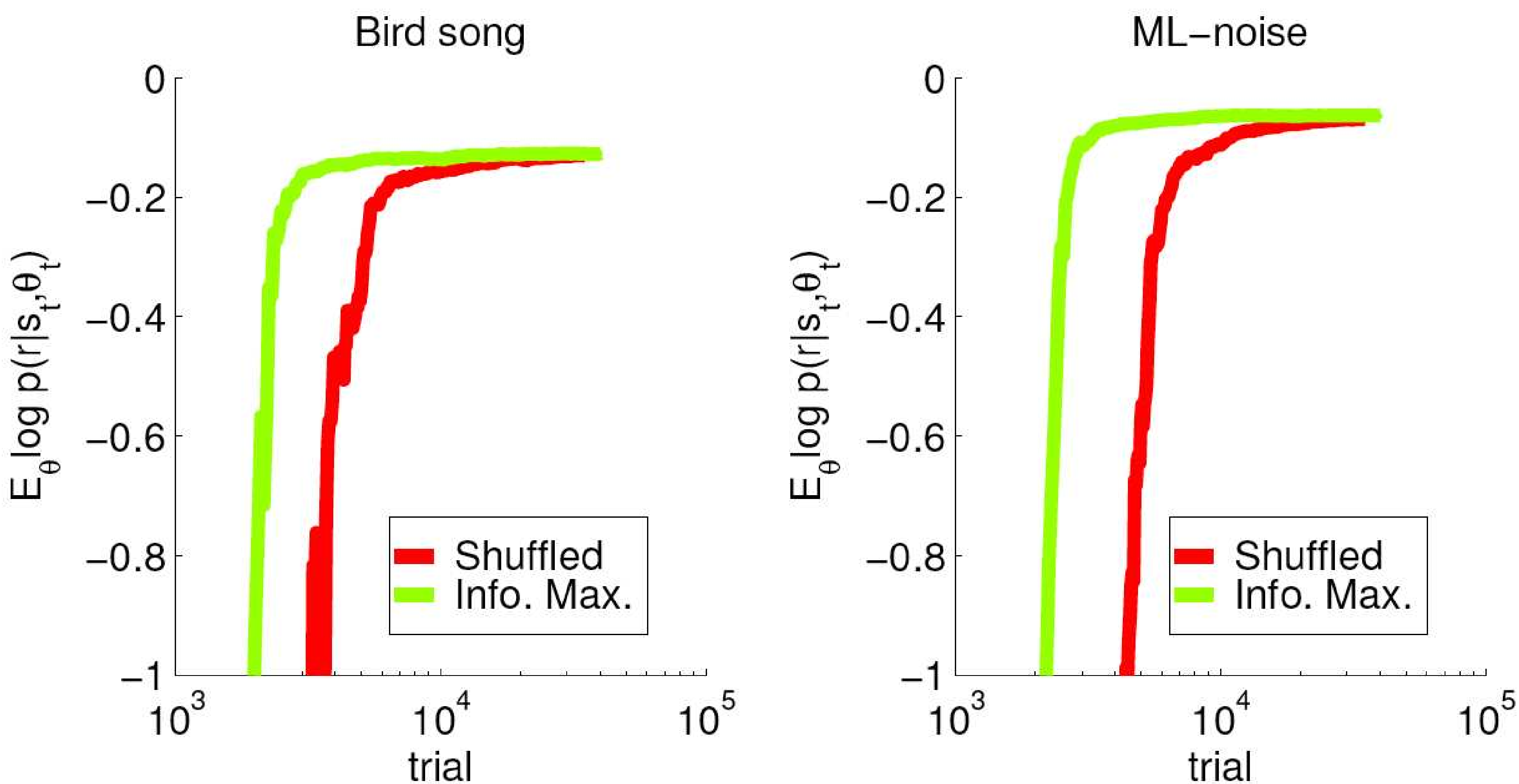


Simulated example



— infomax can be an order of magnitude more efficient.

Application to real data: choosing an optimal stimulus sequence



— stimuli chosen from a fixed pool; greater improvements expected if we can choose arbitrary stimuli on each trial.

Handling nonstationary parameters

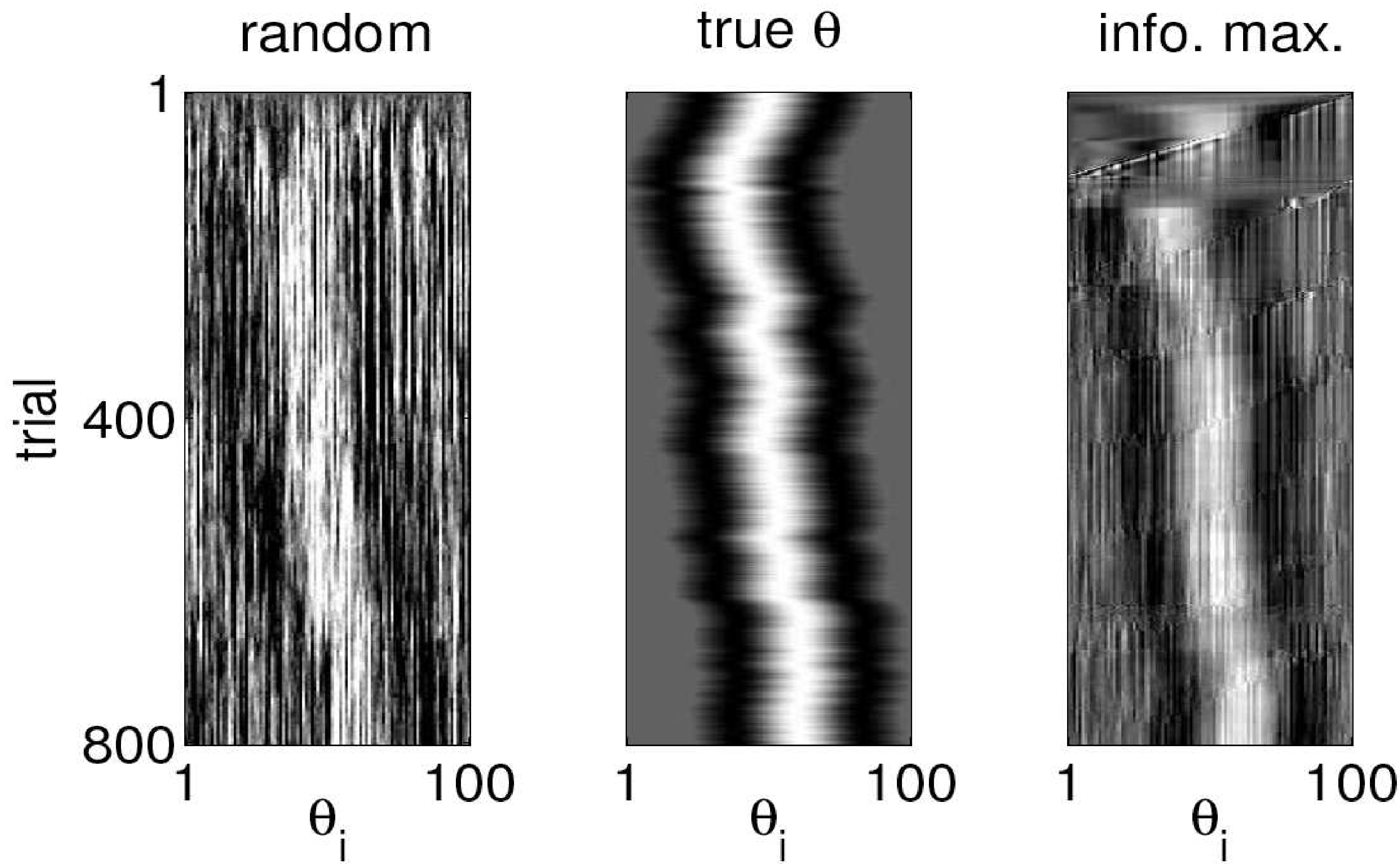
Various sources of nonsystematic nonstationarity:

- Plasticity/adaptation
- Changes in arousal / attentive state
- Changes in health / excitability of preparation

Solution: allow diffusion in parameter θ (Czanner et al., 2008; Lewi et al., 2009):

$$\vec{\theta}_{N+1} = \vec{\theta}_N + \epsilon; \quad \epsilon \sim \mathcal{N}(0, Q)$$

Simulation: nonstationary parameters



Conclusion

GLM framework leads to tractable methods for:

- estimating STRFs including spike-history effects
- optimal decoding
- optimal stimulus design
- nonstationarity tracking.

— Strong potential for applications in birdsong system.

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