Efficient hierarchical receptive field estimation in simultaneously-recorded neural populations

Kamiar Rahnama Rad, Carl Smith, Timothy Machado, and Liam Paninski

Department of Statistics
Columbia University

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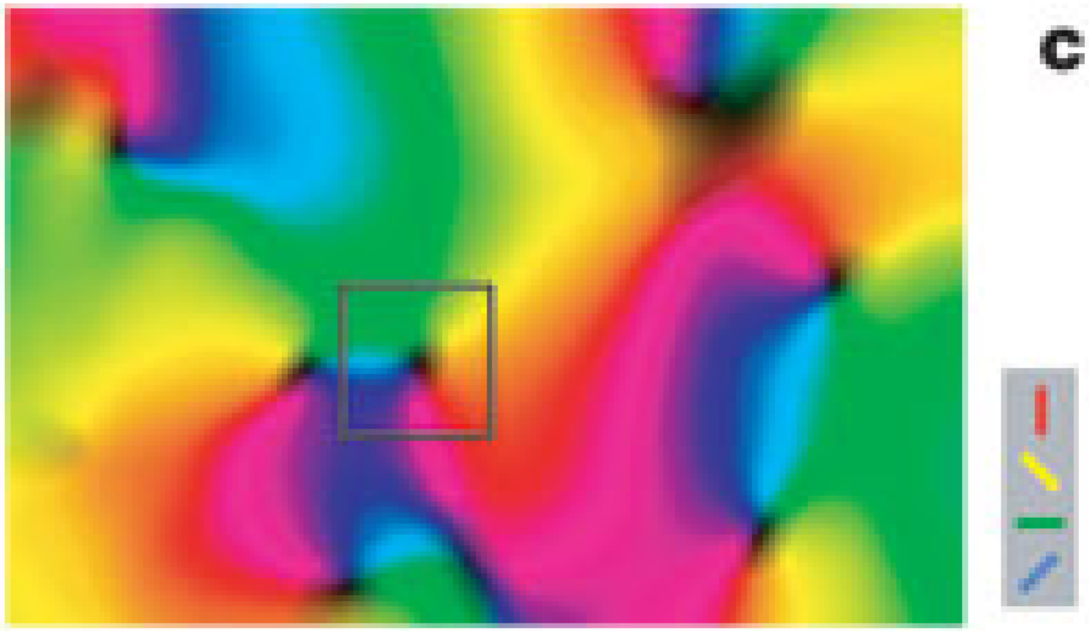
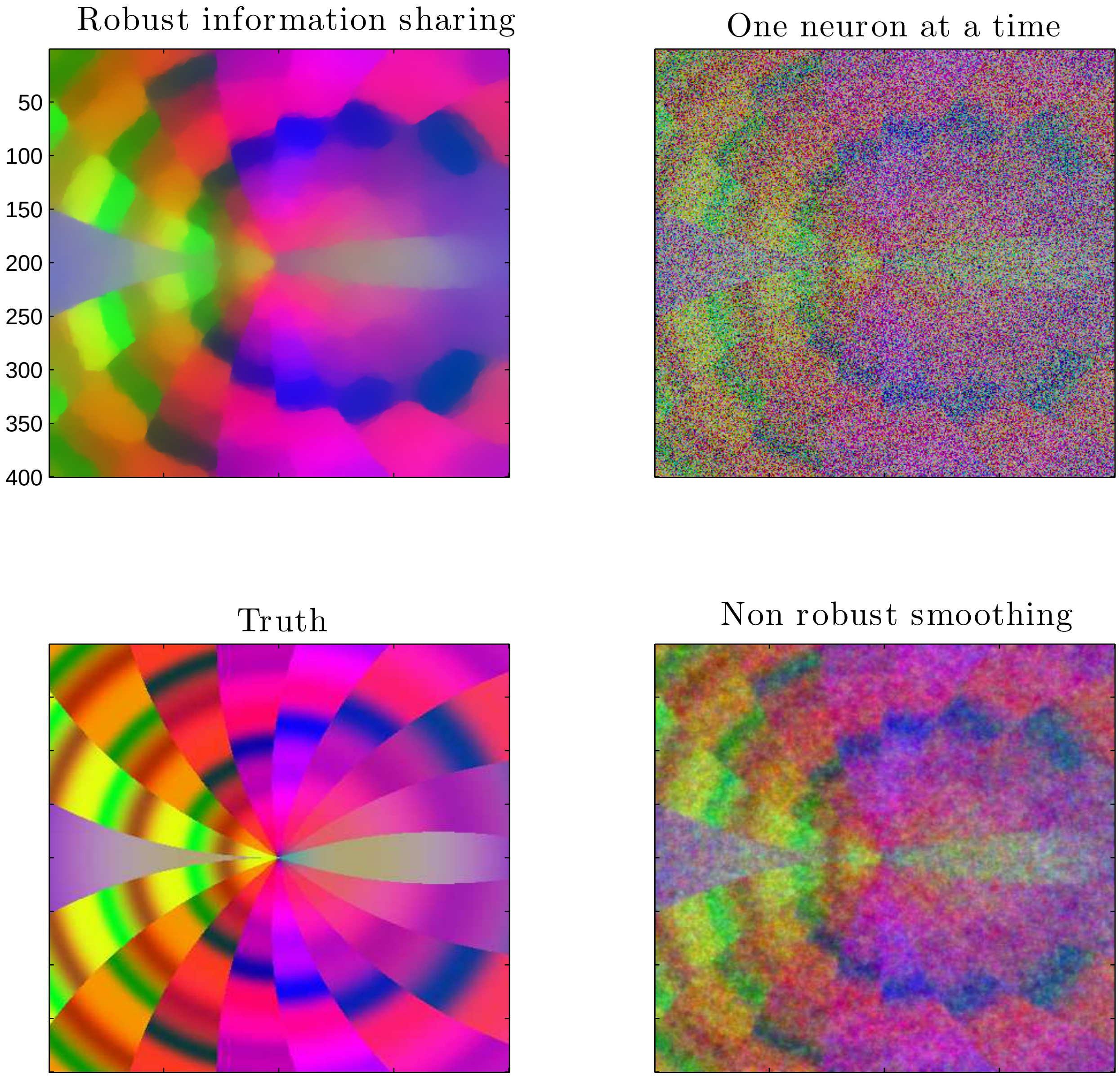
Representation of the visual environment in the brain



Orientation maps

- vary continuously across the cortical surface
- are punctuated by occasional jumps or discontinuities



- 160000 neurons
- 2 spikes per neuron

Neural encoding model

- Inhomogeneous point process with history dependence and coupling

$$
\begin{aligned}
r_{\ell,t} &\sim Poiss\left[\lambda_{\ell,t} dt\right] \\
\lambda_{\ell,t} &= f\left[(X_\ell \theta_\ell)_t + \sum_{\ell'} r_{\ell',t} * h_{\ell,\ell',t}\right]
\end{aligned}
$$

- θ_ℓ is the RF of neuron ℓ
- $(X_\ell \theta_\ell)_t$ is the projection of the stimulus onto the RF of neuron ℓ at time t

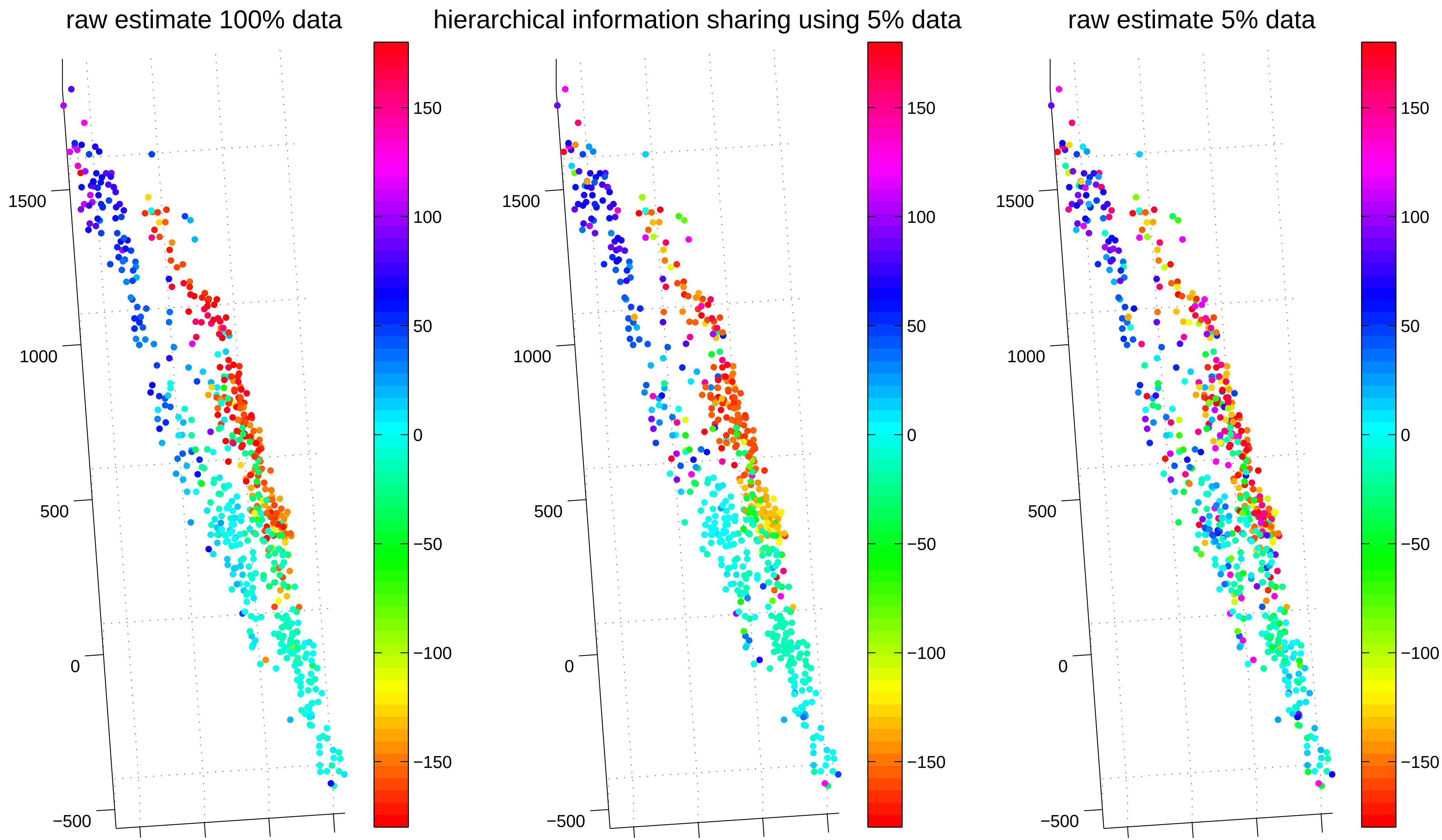
Hierarchical and robust joint estimation of the RF map

Maximum a posteriori estimate by maximizing

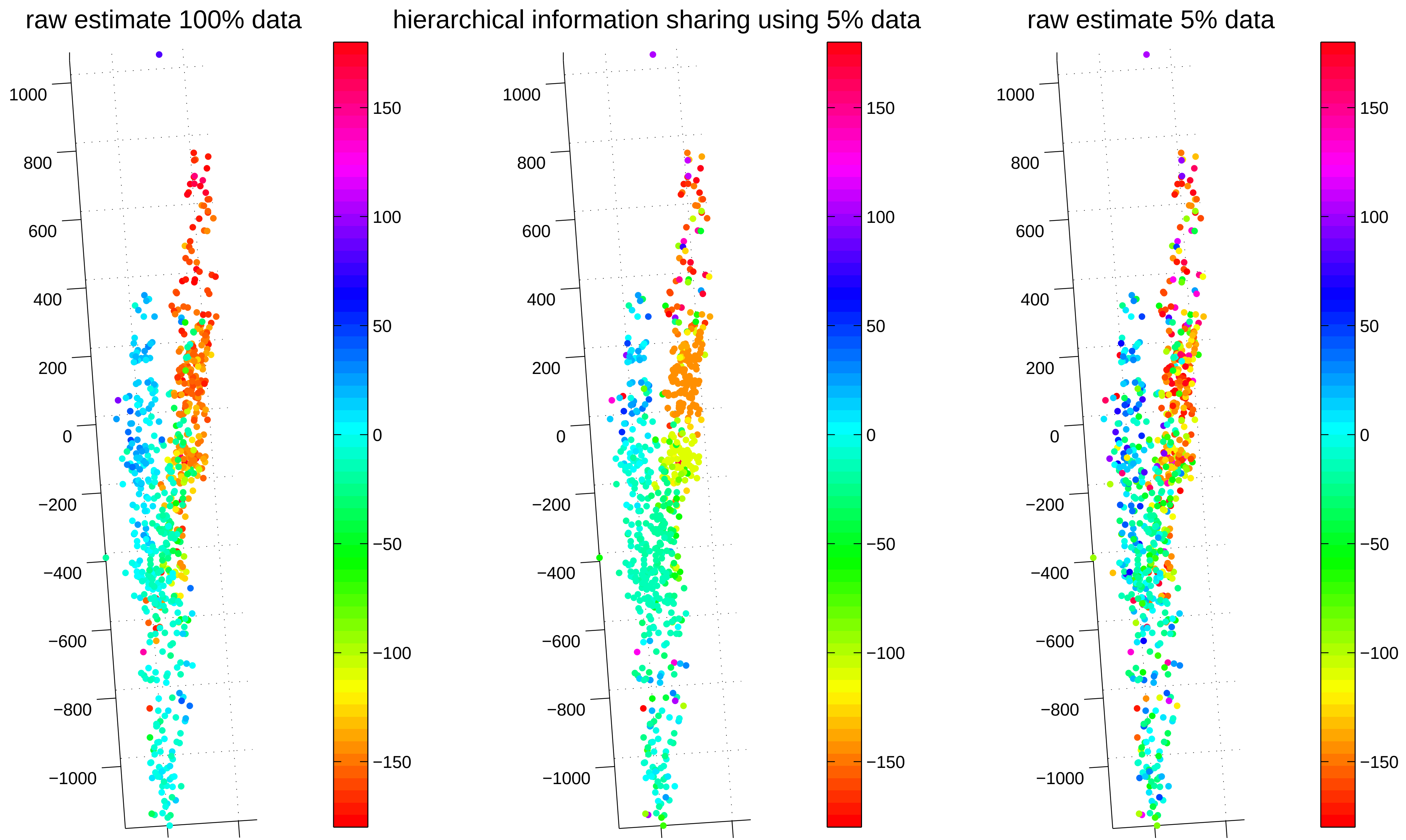
$$\log \Pr(\text{data}|\theta) - \lambda \sum_{\ell} \left\| D_\ell \theta \right\|_2 \quad D_\ell \theta = \begin{bmatrix} \theta_\ell - \theta_{\ell'} \\ \theta_\ell - \theta_{\ell_{\prime}} \end{bmatrix}$$

- neuron ℓ is located at (i, j)
- neuron ℓ' is located at $(i + 1, j)$
- neuron $\ell_{\prime}$ is located at $(i, j + 1)$

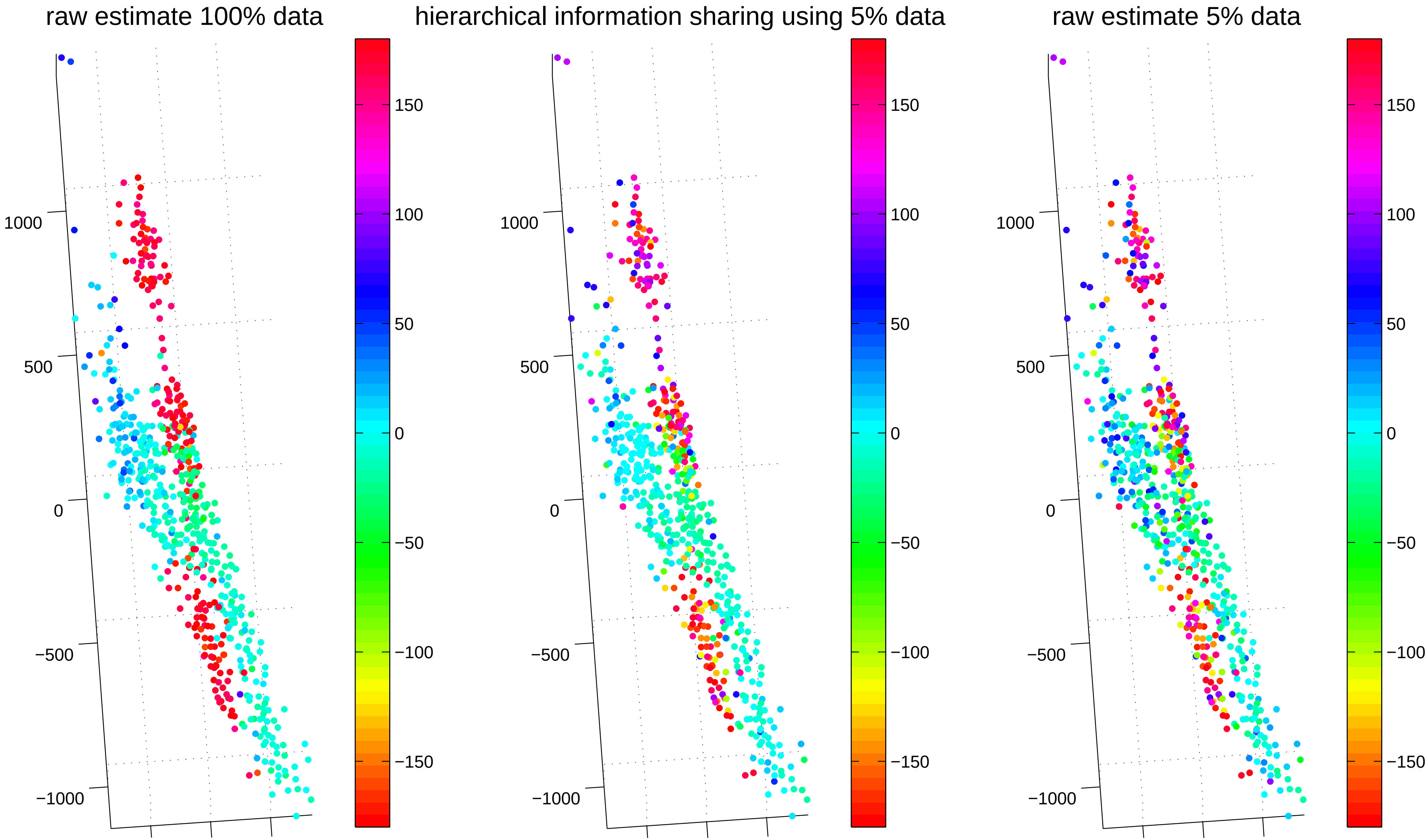
- Estimating receptive fields (or motor preferences) one neuron at a time is highly suboptimal.
- The precision of the border between functional maps can not be resolved unless the smoother is equipped with a right mix of prior information.



Phasic tuning at single-cell resolution



Phasic tuning at single-cell resolution



Phasic tuning at single-cell resolution

- $t_1, \cdots, t_L$ are spike times
- $\alpha_1 = \omega t_1, \cdots, \alpha_L = \omega t_L$ are noisy phases at spikes
- unknown phasic tuning ϕ
-

$$\begin{aligned} r \exp(i\alpha) &= \rho \exp(i\phi) + \text{noise} \\ \text{noise} &= \mathcal{N}\left(0, \sigma^2\right) + i\mathcal{N}\left(0, \sigma^2\right) \end{aligned}$$

Hierarchical and robust joint estimation of the RF map

- Maximum a posteriori estimate by maximizing

$$\log \Pr(\text{data}|\{\phi_\ell\}) - \lambda \sum_{\ell} \sum_{\ell' \in N_\ell} \left\| \begin{bmatrix} \rho_\ell \cos\phi_\ell - \rho_{\ell'} \cos\phi_{\ell'} \\ \rho_\ell \cos\phi_\ell - \rho_{\ell,} \cos\phi_{\ell,} \end{bmatrix} \right\|_2$$

- N_ℓ is the set of all neurons near to neuron ℓ

Mouse spinal cord

- Isolated neonatal mouse spinal cord contains neural circuits that can generate ordered patterns of periodic population activity
- Efficient characterization of the precise structure of phasic tuning at single-cell resolution
- Motor neuron activity was measured using large-scale, cellular resolution calcium imaging across hundreds of identified motor neurons
- See: II-63 Large-scale optical imaging reveals structured network output in isolated spinal cord. T. Machado, L. Paninski, T.M. Jessell

- Optimization has no non-global local minima
- Shares information across neurons: nearby neurons often have similar receptive fields
- Robust and adaptive: allows for large occasional breaks or outliers, contrary to previous work [1, 2].
- Posterior confidence intervals via Gibbs sampling, and scale mixtures
- Newton-Raphson iterations are fast: $O(d \log d)$ time; d = number of cells $\times$ dimensionality of each receptive field

Conlusion

- Estimating RFs one neuron at a time is highly inefficient
- Robust and adaptive information sharing can decreases the duration of the experiment up to 95%
- Adaptive experiment design can be done using posterior confidence intervals
- Hierarchical robust information sharing across neurons can scale to hundreds of thousands of simultaneously recorded neurons

Appendix

Probabilistic modeling

- Likelihood: $\ell = (i, j)$, $\theta_\ell \in \mathbb{R}^m$,

$$\begin{bmatrix} \vdots \\ r_\ell \\ \vdots \end{bmatrix} = \begin{bmatrix} \vdots \\ \theta_\ell \\ \vdots \end{bmatrix} + \sigma \begin{bmatrix} \vdots \\ \epsilon_\ell \\ \vdots \end{bmatrix}$$

- Prior:

$$\begin{aligned} \theta | \sigma^2, \tau_1^2, \cdots, \tau_{n^2}^2 &\sim \mathcal{N}(0, \sigma^2 C_\tau) \\ C_\tau^{-1} &= \sum_\ell \tau_\ell^{-2} D_\ell^T D_\ell = D^T \Gamma_\tau D \end{aligned}$$

$$\sigma^2, \tau_1^2, \cdots, \tau_{n^2}^2 \sim \pi(\sigma^2) d\sigma^2 \times \prod_{p=1}^{n^2} \frac{\lambda^2}{2} e^{-\lambda^2 \tau_p^2 / 2} d\tau_p^2 .$$

Posterior distribution via block Gibbs sampling

$$\theta|\sigma^2, \tau_1^2, \cdots, \tau_{n^2}^2, r \sim \mathcal{N}(\eta, C)$$

$$\eta = \left(I + D^T \Gamma_\tau D\right)^{-1} r$$

$$C = \sigma^2 \left(I + D^T \Gamma_\tau D\right)^{-1}$$

Posterior distribution via block Gibbs sampling

$$\sigma^2 | r, \theta, \tau_1^2, \cdots, \tau_{n^2}^2 \sim \pi(\sigma^2) d\sigma^2 \times \Gamma^{-1}(\alpha, \beta)$$

$$\Gamma^{-1}(\alpha, \beta) = \beta^\alpha x^{-\alpha-1} \frac{e^{-\beta/x}}{\Gamma(\alpha)}$$

$$\begin{aligned} \alpha &:= n^2 - 1, \\ \beta &:= \frac{1}{2}\|r - \theta\|^2 + \frac{1}{2}\theta^T C_\tau^{-1} \theta. \end{aligned}$$

Posterior distribution via block Gibbs sampling

$$\frac{1}{\tau_\ell^2}\Big| r, \theta, \sigma^2 \sim IG(\mu_\ell, \lambda_\ell).$$

where

$$\mu_\ell = \frac{\lambda\sigma}{\|D_\ell\theta\|_2}, \quad \lambda_\ell = \lambda^2,$$

$$IG(\mu, \lambda) = \sqrt{\frac{\lambda}{2\pi}} x^{-3/2} \exp\left\{-\frac{\lambda(x-\mu)^2}{2\mu^2 x}\right\}$$

Posterior distribution via block Gibbs sampling

$\theta|\sigma^2, \tau_1^2, \cdots, \tau_{n^2}^2, r \sim \mathcal{N}(\eta, C)$ where

$$\eta = \left(I + D^T \Gamma_\tau D\right)^{-1} r,$$
$$C = \sigma^2 \left(I + D^T \Gamma_\tau D\right)^{-1}$$

$$\sigma\left(I + D^T \Gamma_\tau D\right)^{-1}\left(\epsilon_1 + D^T \Gamma_\tau^{1/2} \epsilon_2\right) \sim \mathcal{N}(0, C)$$

References

[1] K. Rahnama Rad and L. Paninski. Efficient estimation of two-dimensional firing rate surfaces via gaussian process methods. *Network: Computation in Neural Systems*, 21:142–168, 2010.

[2] J.H. Macke, S. Gerwinn, L.E. White, M. Kaschube, and M. Bethge. Bayesian estimation of orientation preference maps. *NIPS*, 2010.