

Mean-field approximations for coupled populations of generalized linear model spiking neurons with Markov refractoriness

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There has recently been a great deal of interest in inferring network connectivity from the spike trains in populations of neurons. One class of models that has proven quite useful is known in the statistics literature as the “generalized linear model” (GLM), variants of which have been applied successfully in the hippocampus, motor cortex, retina, and cultured cortical slice. In its simplest form, this model incorporates both stimulus-dependence terms and direct coupling terms between each observed neuron in the network; fitting the model parameters leads to an inhomogeneous (stimulus-driven), nonlinear coupled spiking model with delay terms. Statistically speaking, the model is attractive due to its explicitly probabilistic nature, and because fitting the model parameters is surprisingly simple: under certain simple conditions, the log-likelihood function with respect to the model parameters is concave and, hence, the maximum likelihood estimation of number of parameters can be easily performed via standard ascent methods.

Once the model parameters are obtained, we are left with an obvious question: what do we do next? One of the key applications of such a network model is to better understand the input-output properties of the network. For example, we would like to be able to predict the mean firing response of the network given a novel input, and to dissect out the impact of the network coupling terms on this stimulus-response relationship (e.g., how does local inhibition impact the stimulus filtering properties of the network?). We would also like to know how the correlation properties of spike trains in the network might depend on the stimulus, and in general how correlations might encode stimulus information. We can in general study these questions via direct Monte Carlo of the network. However, simulation of a large scale probabilistic spiking network is computationally expensive, since we often need to draw many samples (i.e., run many simulated trials) in order to compute the quantities of interest to the desired precision. More importantly, numerical simulation often provides limited analytical insight into the mechanisms underlying the observed phenomena.

The goal of this study is to investigate how much of the behavior of these GLM networks can be understood using standard analytical “mean-field” approximations. In particular, we develop analytically-tractable approximations for the mean firing rates of the network given novel stimuli, as well as the auto- and cross-correlation and input-output filtering properties of these networks. These approximations are valid when the network coupling terms are small, and lead to deterministic ordinary differential equations that are much easier to analyze than direct Monte Carlo simulation of the network activity. However, in the case of strong refractory effects, these mean-field approximations become inaccurate, since the spike-history terms in the generalized linear model must be large to induce strong refractoriness, and this pushes our approximations beyond their region of accuracy. Therefore we introduce a new model, a generalized linear model with Markovian refractoriness. This new model has several advantages in this setting: it captures strong refractoriness, retains all of the easy fitting properties of the standard generalized linear model, and leads to much more accurate mean field approximations. We validate our mathematical analysis with a variety of simulated examples.

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