An expectation-maximization Fokker-Planck algorithm for the noisy integrate-and-fire model

Maxim Nikitchenko[1] **and Liam Paninski**[1,2]

[1]Center for Theoretical Neurosci and [2]Dept of Statistics, Columbia University

We develop an expectation-maximization (EM) algorithm for a class of noisy integrate-and-fire models, in which the voltage evolves according to the linear stochastic differential equation $dV(t) = (-g(t)V(t) + I(t))dt + \sigma dB_t$, with B_t standard Brownian motion, and $V(t)$ is reset after each spike (Paninski et al., 2004; Pillow et al., 2005). The functions $I(t)$ and $g(t)$ (the input current and membrane conductance, respectively) are given by $I(t) = \sum a_i I_i(t)$ and $g(t) = \sum b_j g_j(t)$, where the functions $I_i(t)$ and $g_j(t)$ are assumed known; thus the unknown parameters that we want to estimate are $\theta = \{\vec{a}, \vec{b}, \sigma\}$. (The reset and threshold voltage parameters may be assumed known, by the usual change of variables.)

We fit the parameters θ via maximum likelihood, given only the functions $\{I_i(t)\}$, $\{g_j(t)\}$, and the observed spike times; no intracellular currents or voltages are observed. We previously (Paninski et al., 2004) described methods for computing the likelihood and directly ascending the likelihood by a hill-climbing procedure. In (Paninski et al., 2005) we proposed an alternate method to compute the likelihood, which facilitates the computation of gradient information. However, the computational complexity of this gradient method scales as $O(d^3)$, where d is the number of time points a given inter-spike interval is divided into (the larger d is, the higher the numerical precision of the calculation). By casting the model as a hidden Markov model in continuous time and space (Paninski, 2006), we may adapt standard EM approaches to derive a method for computing likelihood gradients that only requires $O(d^2)$ time (Salakhutdinov et al., 2003). In addition, the EM algorithm provides a good "warm start" for optimization via conjugate gradient ascent. The E-step of the algorithm requires the solution of a forward and backward Fokker-Planck equation with time-dependent coefficients; we derive an efficient and unconditionally stable algorithm for solving this partial differential equation.

References

Paninski, L. (2006). The spike-triggered average of the integrate-and-fire cell driven by Gaussian white noise. *Neural Computation*, 18:2592–2616.

Paninski, L., Haith, A., Pillow, J., and Williams, C. (2005). Improved numerical methods for computing likelihoods in the stochastic integrate-and-fire model. *Comp. Sys. Neur. '05*.

Paninski, L., Pillow, J., and Simoncelli, E. (2004). Maximum likelihood estimation of a stochastic integrate-and-fire neural model. *Neural Computation*, 16:2533–2561.

Pillow, J., Paninski, L., Uzzell, V., Simoncelli, E., and Chichilnisky, E. (2005). Accounting for timing and variability of retinal ganglion cell light responses with a stochastic integrate-and-fire model. *Journal of Neuroscience*, 25:11003–11013.

Salakhutdinov, R., Roweis, S. T., and Ghahramani, Z. (2003). Optimization with EM and expectation-conjugate-gradient. *International Conference on Machine Learning*, 20:672–679.