A Numerically Efficient Approach for Constructing Reach-Trajectories Conditioned on Target

Jayant E. Kulkarni[1], **Liam Paninski**[1,2]

Center for Theoretical Neuroscience and [2]Dept. of Statistics, Columbia Univ.

The problem of decoding the neural signal to predict a behavioral variable of interest has received much attention from the neuroscience community. The accuracy of state-estimation methods for decoding trajectories can be improved by appropriately restricting the space of possible states. In this work we focus on constructing trajectories, for reaching movements, conditioned on known target states. We show how previous approaches $[1, 2]$ can be extended and improved, by deriving results for computing the priors in a numerically efficient manner, enabling real-time implementation.

In our framework, we model the free-reach equation of the arm by a linear state equation with gaussian noise, $d\vec{x} = Rxdt + WdB_t$, with B_t standard Brownian motion. We observe $\vec{y} = K\vec{x}_T + \epsilon$, where $\epsilon \sim \mathcal{N}(0, M)$, with ϵ independent of $\{\vec{x}(T)\}$. To construct the smoothed trajectory conditioned on the entire data we follow the approach proposed in $[3]$. The smoothed state at a point in time is obtained by optimally melding the output of two filters running forward and backwards in time, with the backwards filter propagating the inverse of the error-covariance matrix. This approach has the advantage that it requires fewer inverses. We can solve for the relevant quantities analytically, using a matrix exponential approach, instead of the slower recursive formulation adopted by [1]. Fig. 1 shows a sample simulation of the single-terminal-state-conditioned-trajectory along with the standard deviation. The terminal state $x(T) = [1 \;\; 1]'$ with the reach motion starting from the origin. $R = \left[\begin{smallmatrix} -.1 & -1 \\ 0 & -1 \end{smallmatrix}\right], W = [0 \;\; 1]'$, and K is taken to be the identity matrix.

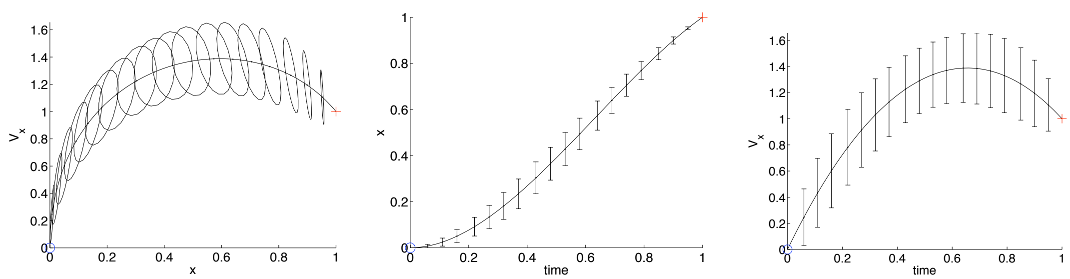

Figure 1: **Sample Simulation for Reach Trajectory Conditioned on Terminal State**

Acknowledgments
J.E.K. would like to acknowledge the Swartz Foundation for a post-doctoral fellowship which supported this research.

References

[1] A state-space analysis for reconstruction of goal-directed movements using neural signals. Srinivasan, L and Eden, U. T. and Willsky, A. S. and Brown, E. N. *Neural Computation* 18:2465–2494, 2006.
[2] Extracting dynamical structure embedded in neural activity. Yu, B.M. and Afsar, A. and Santhanam, G. and Ryu, S. and Shenoy, K. *NIPS*, 2005.
[3] The optimum linear smoother as a combination of two optimum linear filters. Fraser, D. C. and Potter, J. E., *IEEE Transactions on Automatic Control* 14(4):387-390, Aug 1969.