Statistics 1211 (4)
Professor Salzman
Homework#4
Solutions – Section 1

Chapter 4.3.28

a. $P(0 \le Z \le 2.17) = \Phi(2.17) - \Phi(0) = .4850$

b. $\Phi(1) - \Phi(0) = .3413$

c. $\Phi(0) - \Phi(-2.50) = .4938$

d. $\Phi(2.50) - \Phi(-2.50) = .9876$

e. $\Phi(1.37) = .9147$

f. $P(-1.75 < Z) + [1 - P(Z < -1.75)] = 1 - \Phi(-1.75) = .9599$

g. $\Phi(2) - \Phi(-1.50) = .9104$

h. $\Phi(2.50) - \Phi(1.37) = .0791$

i. $1 - \Phi(1.50) = .0668$

j. $P(|Z| \le 2.50) = P(-2.50 \le Z \le 2.50) = \Phi(2.50) - \Phi(-2.50) = .9876$

Chapter 4.3.29

a. .9838 is found in the 2.1 row and the .04 column of the standard normal table so c = 2.14.

b. $P(0 \le Z \le c) = .291 \Rightarrow \Phi(c) = .7910 \Rightarrow c = .81$

c. $P(c \le Z) = .121 \Rightarrow 1 - P(c \le Z) = P(Z < c) = \Phi(c) = 1 - .121 = .8790 \Rightarrow c = 1.17$

d. $P(-c \le Z \le c) = \Phi(c) - \Phi(-c) = \Phi(c) - (1 - \Phi(c)) = 2\Phi(c) - 1$
$\Rightarrow \Phi(c) = .9920 \Rightarrow c = .97$

e. $P(c \le |Z|) = .016 \Rightarrow 1 - .016 = .9840 = 1 - P(c \le |Z|) = P(|Z| < c)$
$= P(-c < Z < c) = \Phi(c) - \Phi(-c) = 2\Phi(c) - 1$
$\Rightarrow \Phi(c) = .9920 \Rightarrow c = 2.41$

Chapter 4.3.36

a. $P(X < 1500) = P(Z < 3) = \Phi(3) = .9987$; $P(X \ge 1000) = P(Z \ge -.33) = 1 - \Phi(-.33) = 1 - .3707 = .6293$

b. $P(1000 < X < 1500) = P(-.33 < Z < 3) = \Phi(3) - \Phi(-.33) = .9987 - .2707 = .7280$

c. From the table, $\Phi(z) = .02 \rightarrow z \approx -2.05 \rightarrow x = 1050 - 2.05(150) = 742.5$ μm. The smallest 2% of droplets are those smaller than 742.5 μm in size

d. P(at least one droplet in 5 that exceeds 1500 μm) = 1 – P(all 5 are less than 1500 μm) = 1 – $(.9987)^5 = 1 - .9935 = .0065$

Chapter 4.3.37

a. P(X = 105) = 0, since the normal distribution is continuous;
P(X < 105) = P(Z < 0.2) = P(Z ≤ 0.2) = Φ(0.2) = .5793;
P(X ≤ 105) = .5793 as well, since X is continuous

b. No, the answer does not depend on μ or σ. For any normal rv, P(|X–μ| > σ) = P(|Z| > 1) = P(Z < –1 or Z > 1) = 2Φ(–1) = 2(.1587) = .3174

c. From the table, Φ(z) = .1% = .001 → z ≈ –3.09 → x = 104 – 3.09(5) = 88.55 mmol/L.
The smallest .1% of chloride concentration values are those less than 88.55 mmol/L

Chapter 4.3.47

The stated condition implies that 99% of the area under the normal curve with μ = 12 and σ = 3.5 is to the left of c – 1, so c – 1 is the 99th percentile of the distribution. Thus c – 1 = μ + σ(2.33) = 20.155, and c = 21.155.

Problem 100 Page 179, parts a-d

a. Clearly f(x) ≥ 0. The c.d.f. is , for x > 0,

$$F(x) = \int_{-\infty}^{x} f(y)dy = \int_{0}^{x} \frac{32}{(y+4)^3}dy = -\frac{1}{2}\cdot\frac{32}{(y+4)^2}\Big]_0^x = 1 - \frac{16}{(x+4)^2}$$

(F(x) = 0 for x ≤ 0.)

Since F(∞) = $\int_{-\infty}^{\infty} f(y)dy = 1$, f(x) is a legitimate pdf.

b. See above

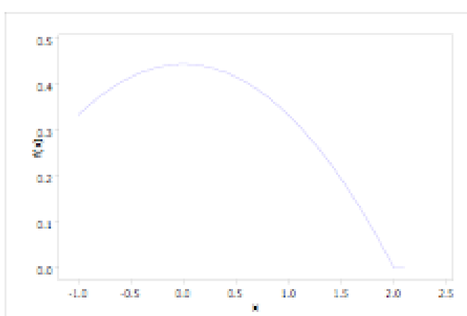
c. $P(2 \le X \le 5) = F(5) - F(2) = 1 - \frac{16}{81} - \left(1 - \frac{16}{36}\right) = .247$

d. $$E(x) = \int_{-\infty}^{\infty} x\cdot f(x)dx = \int_{-\infty}^{\infty} x\cdot\frac{32}{(x+4)^3}dx = \int_0^{\infty}(x+4-4)\cdot\frac{32}{(x+4)^3}dx$$

$$= \int_0^{\infty}\frac{32}{(x+4)^2}dx - 4\int_0^{\infty}\frac{32}{(x+4)^3}dx = 8 - 4 = 4$$

Problem 107 Page 180, parts a-c

a.



b. F(x) = 0 for x < –1 or = 1 for x > 2. For –1 ≤ x ≤ 2,

$$F(x) = \int_{-1}^{x}\frac{1}{9}\left(4 - y^2\right)dy = \frac{1}{9}\left(4x - \frac{x^3}{3}\right) + \frac{11}{27}$$

c. The median is 0 iff F(0) = .5. Since F(0) = $\frac{11}{27}$, this is not the case. Because $\frac{11}{27}$ < .5, the median must be greater than 0.

Problem 123 Page 182

a. $F_X(x) = P\left(-\frac{1}{\lambda}\ln(1-U) \le x\right) = P(\ln(1-U) \ge -\lambda x) = P(1-U \ge e^{-\lambda x})$

$= P(U \le 1-e^{-\lambda x}) = 1-e^{-\lambda x}$ since $F_U(u) = u$ (U is uniform on [0, 1]). Thus X has an exponential distribution with parameter λ.

b. By taking successive random numbers $u_1, u_2, u_3, \ldots$ and computing $x_i = -\frac{1}{10}\ln(1-u_i)$, … we obtain a sequence of values generated from an exponential distribution with parameter $\lambda = 10$.