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ANSWER KEY: HOMEWORK 5

5.1.7) $X = \text{cars}$
 $Y = \text{buses}$

a) $P(X=1, Y=1) = .030$

b) $P(\text{at most 1 car and at most 1 bus}) =$
 $P(X=0, Y=0) + P(X=0, Y=1) + P(X=1, Y=0) +$
 $P(X=1, Y=1)$
 $= .025 + .015 + .050 + .030$
 $= 0.12$

c) $P(X=1) = P(X=1, Y=0) + P(X=1, Y=1) + P(X=1, Y=2)$
 $= .05 + .03 + .02$
 $= .1$

$P(Y=1) = P(X=0, Y=1) + P(X=1, Y=1) + P(X=2, Y=1)$
 $+ P(X=3, Y=1) + P(X=4, Y=1) + P(X=5, Y=1)$
 $= .015 + .03 + .075 + .09 + .06 + .03$
 $= 0.3$

d) $P(\text{overflow}) = P(X=3, Y=1) + P(X=3, Y=2) +$
 $P(X=4, Y=1) + P(X=4, Y=2) +$
 $P(X=5, Y=1) + P(X=5, Y=2) +$
 $P(X=0, Y=2) + P(X=1, Y=2) + P(X=2, Y=2)$
 $= .09 + .06 + .06 + .04 + .03 + .02 +$
 $.01 + .02 + .05$
 $= 0.38$

$$\begin{aligned}
 \overset{-x}{x+y \leq 2} \quad c. - P(X+Y \leq 2) &= \int_0^2 \int_0^{2-x} e^{-x-y} dy dx \\
 y \leq 2-x &= \int_0^2 (e^{-x} - e^{-x-(2-x)}) dx \\
 &= \int_0^2 e^{-x} - e^{-2} dx \\
 &= 1 - e^{-x} - 2e^{-x} \Big|_0^2 \\
 &= 1 - e^{-2} - 2e^{-2} \\
 &= 1 - 3e^{-2} = .594
 \end{aligned}$$

$$\begin{aligned}
 d. - P(X+Y \leq 1) &= \int_0^1 \int_0^{1-x} e^{-x-y} dy dx \\
 &= \int_0^1 (e^{-x} - e^{-1}) dx = 1 - 2e^{-1} \\
 &= .26
 \end{aligned}$$

$$\begin{aligned}
 P(1 \leq X+Y \leq 2) &= P(X+Y \leq 2) - P(X+Y \leq 1) \\
 &= .594 - .26 \\
 &= .334
 \end{aligned}$$

c. - When a and c have different signs, then $\text{Corr}(aX+b, cY+d) = -\text{Corr}(X, Y)$

37.) a. -

Size (oz)		25	40	65
		.2	.5	.3
25	.2	.04	.1	.06
40	.5	.1	.25	.15
65	.3	.06	.15	.09

\bar{x}	25	40	65	32.5	52.5	45
$p(\bar{x})$.04	.25	.09	.2	.3	.12

$$\begin{aligned} \mu = E(\bar{x}) &= (25)(.04) + (40)(.25) + (65)(.09) + (32.5)(.2) \\ &\quad + (52.5)(.3) + (45)(.12) \\ &= 44.5 \end{aligned}$$

Since $s^2 = \frac{1}{n-1} \sum (x_i - \bar{x})^2$

b.) $s^2 = 0$	$p(s^2) = .38$
$s^2 = 112.5$	$p(s^2) = .2$
$s^2 = 312.5$	$p(s^2) = .3$
$s^2 = 800$	$p(s^2) = .12$

$$\begin{aligned} E(s^2) &= (0)(.38) + (112.5)(.2) + (312.5)(.3) + \\ &\quad (800)(.12) \\ &= 212.25 \end{aligned}$$

5.3.41) $X = \#$ of packages mailed

$a - \bar{x}$	\bar{x}	1	1.5	2	2.5	3	3.5	4
$P(\bar{x})$.16	.24	.25	.2	.1	.04	.01

b. $- P(\bar{x} \leq 2.5) = .2 + .25 + .24 + .16 = .85$

$c - R$	0	1	2	3
$P(R)$.3	.4	.22	.08

d. $- P(\bar{x} \leq 1.5) = P(1, 1, 1, 1) + P(2, 1, 1, 1) + P(1, 2, 1, 1)$
 $+ P(1, 1, 2, 1) + P(1, 1, 1, 2) + P(2, 2, 1, 1)$
 $+ P(2, 1, 2, 1) + P(2, 1, 1, 2) + P(1, 1, 2, 2)$
 $+ P(3, 1, 1, 1) + P(1, 3, 1, 1) + P(1, 1, 3, 1)$
 $+ P(1, 1, 1, 3)$
 $= (.4 \cdot .4 \cdot .4 \cdot .4) + 4(.4)^3 + 6(.4)^2(.3)^2 +$
 $4(.4)^2(.2)^2$
 $= .24$