Name:

Directions: Write your answers out completely and include all work on the pages given. The test is exactly 1 hour. Please observe this limit. NO calculators may be used and NO consultation of any written or electronic material is allowed. INCLUDE ALL CALCULATIONS FOR PARTIAL CREDIT.

Remember the central limit theorem says that the distribution of the sample mean from a population with mean μ and variance σ^2 can be approximated by a normal distribution with mean μ and variance σ^2/n. Also, remember that the probability a standard normal is within 1 standard deviation of 0 is .68.

Good luck!

1 (10)	
2 (15)	
3 (35)	
4 (25)	
5 (30)	

1. (10 points)

a.) Write the density function for a normal random variable with mean μ and variance σ^2 (5 points).

b.) what is the maximal value of $f(x)$ in terms of μ and σ (5 points)?

2. (15 points) Give an example of two random variables X and Y where X takes only two values, Y takes only three values and where X and Y are not independent, yet have zero correlation. Recall that the covariance between X and Y is $E(XY) - E(X)E(Y)$, and $Cor(X, Y) = \frac{Cov(X,Y}{\sigma_X \sigma_Y}$.

3. (35 points) A total of 10 people are gathered in a room, i.e. each person has a distinct number $1, \ldots, 10$. 6 out of 10 favor Obama and the other 4 out of 10 favor Clinton. You sample 3 people in the room with replacement by randomly selecting a number between 1 and 10, with replacement. Let X_i be the random variable representing the preference of the i^{th} person you sample in the following way. X_i is 1 if the person favors Obama and zero otherwise.

a) Name the distribution of X_i (5 points).

b) What is $E(X_i)$ (5 points)?

c) Compute $var(X_i)$ (5 points).

d) The above set-up produces a simple random sample. What is the probability that you will find $\bar{X} = 0$ (5 points)?

e) Find all possible values of $\bar{X}$ and write their probabilities (10 points).

f) Draw the probability mass function for $\bar{X}$ or write it down explicitly and completely. (5 points).

4. (25 points) Suppose that X, the number of miles a bicycle rider rides each year is $Uniform[0, 10000]$ (that is, the density $f(x)$ for X is $\frac{1}{10000}$ if $0 \leq x \leq 10000$ and 0 otherwise). In what follows, suppose you have a simple random sample from a distribution X.

a) Calculate the variance of X (10 points).

b) If you prefer, you may use σ to denote the variance of X in what follows: If 36 people are sampled, what is the approximate distribution of $\bar{X}$? (Write down in words and also write down the density function approximating $\bar{X}$) (5 points).

c) If 100 people are sampled, what is the approximate distribution of $\bar{X}$? (Write down in words and also write down the density function approximating $\bar{X}$) (5 points).

d) In the set-up of b), find the constant a so that with probability .68, the difference of the sample mean and the true mean is no more than a in absolute value (5 points).

5. (30 points) Suppose that $f(x, y) = kxy$ is a joint density function representing the joint distribution of a random variable X and Y (X, Y are the fraction of cell phone minutes per hour two college roommates spend on the phone, so $0 \leq X \leq 1$ and $0 \leq Y \leq 1$). k is a constant which you are not given.

1. What is the value of k (5 points)?
2. Compute $Cov(X, Y)$ (5 points).
3. What is the expectation of X (5 points) ?
4. Compute the probability that the sum of X and Y is less than $\frac{1}{2}$ (5 points)?
5. Suppose instead of the set-up of the problem, that X and Y are random variables that are not independent of each other. Give an example of $f(x, y)$ where X and Y are dependent and explain rigorously why this is is true (10 points).