Statistics 1211
Professor Salzman
Homework 6 Solutions

Chapter 5.3.40

a. Possible values of M are: 0, 5, 10. M = 0 when all 3 envelopes contain 0 money, hence p(M = 0) = $(.5)^3$ = .125. M = 10 when there is a single envelope with $10, hence p(M = 10) = 1 – p(no envelopes with $10) = 1 – $(.8)^3$ = .488.
p(M = 5) = 1 – [.125 + .488] = .387.

M	0	5	10
p(M)	.125	.387	.488

An alternative solution would be to list all 27 possible combinations using a tree diagram and computing probabilities directly from the tree.

b. The statistic of interest is M, the maximum of x_1, x_2, or x_3, so that M = 0, 5, or 10. The population distribution is a s follows:

x	0	5	10
p(x)	1/2	3/10	1/5

Write a computer program to generate the digits 0 – 9 from a uniform distribution. Assign a value of 0 to the digits 0 – 4, a value of 5 to digits 5 – 7, and a value of 10 to digits 8 and 9. Generate samples of increasing sizes, keeping the number of replications constant and compute M from each sample. As n, the sample size, increases, p(M = 0) goes to zero, p(M = 10) goes to one. Furthermore, p(M = 5) goes to zero, but at a slower rate than p(M = 0).

Chapter 5.4.49

a. 11 P.M. – 6:50 P.M. = 250 minutes. With $T_0 = X_1 + \ldots + X_{40}$ = total grading time, $\mu_{T_0} = n\mu = (40)(6) = 240$ and $\sigma_{T_0} = \sigma\sqrt{n} = 37.95$, so $P(T_0 \le 250) \approx$

$$P\left(Z \le \frac{250-240}{37.95}\right) = P(Z \le .26) = .6026$$

b. $$P(T_0 > 260) = P\left(Z > \frac{260-240}{37.95}\right) = P(Z > .53) = .2981$$

Chapter 5.4.51

$X \sim N(10,4)$. For day 1, n = 5

$$P(\overline{X} \le 11) = P\left(Z \le \frac{11-10}{2/\sqrt{5}}\right) = P(Z \le 1.12) = .8686$$

For day 2, n = 6

$$P(\overline{X} \le 11) = P\left(Z \le \frac{11-10}{2/\sqrt{6}}\right) = P(Z \le 1.22) = .8888$$

For both days,

$$P(\overline{X} \le 11) = (.8686)(.8888) = .7720$$

Chapter 5.4.54

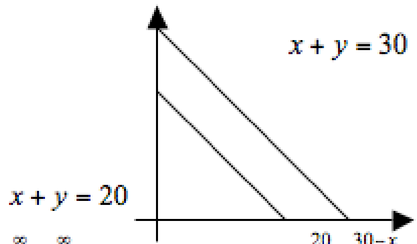
a. $\mu_{\overline{X}} = \mu = 2.65$, $\sigma_{\overline{x}} = \frac{\sigma_x}{\sqrt{n}} = \frac{.85}{5} = .17$

$$P(\overline{X} \le 3.00) = P\left(Z \le \frac{3.00-2.65}{.17}\right) = P(Z \le 2.06) = .9803$$

$$P(2.65 \le \overline{X} \le 3.00) = P(\overline{X} \le 3.00) - P(\overline{X} \le 2.65) = .4803$$

b. $P(\overline{X} \le 3.00) = P\left(Z \le \frac{3.00-2.65}{.85/\sqrt{n}}\right) = .99$ implies that $\frac{.35}{85/\sqrt{n}} = 2.33$, from which n = 32.02. Thus n = 33 will suffice.

Chapter 5.77



a. $1 = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)dxdy = \int_{0}^{20}\int_{20-x}^{30-x} kxydydx + \int_{20}^{30}\int_{0}^{30-x} kxydydx$

$= \frac{81{,}250}{3} \cdot k \Rightarrow k = \frac{3}{81{,}250}$

b. $f_X(x) = \begin{cases} \int_{20-x}^{30-x} kxydy = k(250x - 10x^2) & 0 \le x \le 20 \\ \int_{0}^{30-x} kxydy = k(450x - 30x^2 + \frac{1}{2}x^3) & 20 \le x \le 30 \end{cases}$

and by symmetry $f_Y(y)$ is obtained by substituting y for x in $f_X(x)$. Since $f_X(25) > 0$, and $f_Y(25) > 0$, but $f(25, 25) = 0$, $f_X(x) \cdot f_Y(y) \neq f(x,y)$ for all x,y so X and Y are not independent.

c. $P(X + Y \le 25) = \int_{0}^{20}\int_{20-x}^{25-x} kxydydx + \int_{20}^{25}\int_{0}^{25-x} kxydydx$

$= \frac{3}{81{,}250} \cdot \frac{230{,}625}{24} = .355$

d. $E(X+Y) = E(X) + E(Y) = 2\left\{\int_{0}^{20} x \cdot k(250x - 10x^2)dx\right.$

$\left. + \int_{20}^{30} x \cdot k(450x - 30x^2 + \frac{1}{2}x^3)dx\right\} = 2k(351{,}666.67) = 25.969$

e. $E(XY) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} xy \cdot f(x,y)dxdy = \int_{0}^{20}\int_{20-x}^{30-x} kx^2y^2dydx$

$+ \int_{20}^{30}\int_{0}^{30-x} kx^2y^2dydx = \frac{k}{3} \cdot \frac{33{,}250{,}000}{3} = 136.4103$, so

$Cov(X,Y) = 136.4103 - (12.9845)^2 = -32.19$, and $E(X^2) = E(Y^2) = 204.6154$, so

$\sigma_x^2 = \sigma_y^2 = 204.6154 - (12.9845)^2 = 36.0182$ and $\rho = \frac{-32.19}{36.0182} = -.894$

f. $Var(X+Y) = Var(X) + Var(Y) + 2Cov(X,Y) = 7.66$

Chapter 5.78

$F_Y(y) = P(\max(X_1, \ldots, X_n) \le y) = P(X_1 \le y, \ldots, X_n \le y) = [P(X_1 \le y)]^n = \left(\frac{y-100}{100}\right)^n$ for $100 \le y \le 200$.

Thus $f_Y(y) = \frac{n}{100^n}(y-100)^{n-1}$ for $100 \le y \le 200$.

$$E(Y) = \int_{100}^{200} y \cdot \frac{n}{100^n}(y-100)^{n-1}\,dy = \frac{n}{100^n}\int_0^{100}(u+100)u^{n-1}\,du$$

$$= 100 + \frac{n}{100^n}\int_0^{100} u^n\,du = 100 + 100\frac{n}{n+1} = \frac{2n+1}{n+1}\cdot 100$$