ANSWER KEY (HOMEWORK 3)
Section 3.3 (29, 41, 42, 44)
Section 3.4 (59, 62, 67)
Section 3.6 (80) + Supplementary Exercises (108)

29.) a.- $E(X) = \sum x\,p(x)$

$$= (0)(.08) + (1)(.15) + (2)(.45) + (3)(.27) + (4)(.05)$$

$$= \boxed{2.06}$$

b.- $V(X) = \sum (x-\mu)^2 p(x)$

$$= (0-2.06)^2(.08) + (1-2.06)^2(.15) + (2-2.06)^2(.45) + (3-2.06)^2(.27) + (4-2.06)^2(.05)$$

$$= \boxed{0.9364}$$

c.- $\sigma_X = \sqrt{V(X)} = \sqrt{0.9364} = \boxed{0.968}$

d.- $V(X) = \left[\sum x^2 p(x)\right] - \mu^2$

$$= \left[(0)^2(.08) + (1)^2(.15) + (2)^2(.45) + (3)^2(.27) + (4)^2(.05)\right] - 2.06^2$$

$$= 0.9364$$

41.) Expression 3.13:

$$V[h(X)] = \sum \{h(x) - E[h(X)]\}^2 \cdot p(x)$$

If $h(x) = aX + b$,

$$h(x) - E[h(x)] = ax + b - (a\mu + b) = a(x-\mu)$$

$$\therefore V[h(X)] = \sum [a(x-\mu)]^2 p(x)$$

$$= \sum [a^2(x^2 - 2\mu x + \mu^2)]\, p(x)$$

$$= \left[a^2 \sum x^2 - a^2 \sum 2\mu x + \sum \mu^2\right] p(x)$$

$$a^2\sum x^2p(x) - a^2 2\mu \sum xp(x) + \sum \mu^2 p(x)$$

$$a^2E(X^2) - a^2 2\mu E(X) + \mu^2 \overset{1}{\sum p(x)}$$

$$a^2E(X^2) - a^2 2\mu E(X) + \mu^2$$

Since $\mu = E(X)$, $$a^2E(X^2) - a^2 2\boxed{[E(X)]^2} + E(X)^2$$

$$a^2[E(X^2) - \boxed{[E(X)]^2}] = a^2\sigma_X^2$$

42) a.- $E(X)^2 = 32.5$

$$E[X(X-1)] = E[X^2 - X] = E(X^2) - E(X)$$

$$27.5 = E(X^2) - 5$$

$$32.5 = E(X^2)$$

b.- $V(X) = E(X^2) - [E(X)]^2$

$$= 32.5 - 5^2 = 7.5$$

c.- $V(X) = E(X^2) - (E(X))^2$

$$V(X) = E(X(X-1)) + E(X) - (E(X))^2$$

44) $P\left(\left|\frac{X-\mu}{\sigma}\right| \geq \frac{k\sigma}{\sigma}\right) \leq \frac{1}{k^2}$

$$P\left(\frac{|X-\mu|}{\sigma} \geq k\right) \leq \frac{1}{k^2}$$

a.- $P\left(\frac{|X-\mu|}{\sigma} \geq 2\right) \leq \frac{1}{2^2} = \frac{1}{4}$

$$P\left(\frac{|X-\mu|}{\sigma} \geq 3\right) \leq \frac{1}{3^2} = \frac{1}{9}$$

$$P\left(\frac{|X-\mu|}{\sigma} \geq 4\right) \leq \frac{1}{4^2} = \frac{1}{16}$$

$$P\left(\frac{|X-\mu|}{\sigma} \geq 5\right) \leq \frac{1}{5^2} = \frac{1}{25}$$

$$P\left(\frac{|X-\mu|}{\sigma} \geq 10\right) \leq \frac{1}{10^2} = \frac{1}{100}$$

b. $\mu = (0)(.1) + (1)(.15) + (2)(.2) + 3(.25) + 4(.2) + 5(.06) + 6(.04)$
$= 2.64$

$E(X^2) = 0^2(0.10) + 1^2(0.15) + 2^2(0.2) + 3^2(0.25) + 4^2(0.2) + 5^2(0.06) + 6^2(0.04)$
$= 9.34$

$\sigma_X^2 = E(X^2) - \mu^2 = 9.34 - 2.64^2 = 2.37$

$\sigma_X = \sqrt{2.37} = 1.54$

$k = 2$:
$\mu - k\sigma = 2.64 - 2(1.54) = -0.44$
$\mu + k\sigma = 2.64 + 2(1.54) = 5.72$
$P(X \leq -0.44 \text{ or } X \geq 5.72) = 0.04$

$k = 3$:
$\mu - k\sigma = 2.64 - 3(1.54) = -1.98$
$\mu + k\sigma = 2.64 + 3(1.54) = 7.26$
$P(X \leq -1.98 \text{ or } X \geq 7.26) \approx 0$

$k = 4$:
$\mu - k\sigma = 2.64 - 4(1.54) = -3.52$
$\mu + k\sigma = 2.64 + 4(1.54) = 8.8$
$P(X \leq -3.52 \text{ or } X \geq 8.8) \approx 0$

59.) X = number of homes among 25 with detectors

$p \geq .8$ if $x \leq 15$

a.- $b(15; 25, .8) = 0.017$

b.- When $p = .7$, $1 - b(15; 25, .7)$
$= 1 - .189$
$= 0.811$

When $p = .6$, $1 - b(15; 25, .6)$
$= 1 - .575$
$= 0.425$

c.- For ~~part~~ (a), the error probability would decrease while it would increase for parts (b) and (c)

62.) a.- If $X \sim Bin(n, p)$, then $V(X) = np(1-p)$. For $V(X) = 0$, p must be 0 or 1.

b.- $V(X) = np(1-p) = np - np^2$
$V'(X) = n - 2np = 0$
$$\frac{n}{2n} = \frac{2np}{2n}$$
$$\boxed{p = \frac{n}{2n} = \frac{1}{2}}$$

(67.) $X \sim Bin(20, 0.5)$

$E(X) = np = (20)(0.5) = 10$

$V(X) = np(1-p) = (20)(0.5)(0.5) = 5$

$\sigma = S(X) = \sqrt{V(X)} = \sqrt{5} \approx 2.24$

$k = 2$:

$\mu - k\sigma = 10 - 2(2.24) = 5.52$

$\mu + k\sigma = 10 + 2(2.24) = 14.48$

$P(X \leq 5.52 \text{ or } X \geq 14.48) = 0.021 + 0.021$

$= 0.042$

$k = 3$:

$\mu - k\sigma = 10 - 3(2.24) = 3.28$

$\mu + k\sigma = 10 + 3(2.24) = 16.72$

$P(X \leq 3.28 \text{ or } X \geq 16.72) = 0.001 + 0.001$

$= 0.002$

$X \sim Bin(20, 0.75)$

$E(X) = np = (20)(.75) = 15$

$V(X) = np(1-p) = (20)(.75)(.25) = 3.75$

$\sigma = S(X) = \sqrt{V(X)} = \sqrt{3.75} \approx 1.94$

$k = 2$:

$\mu - k\sigma = 15 - 2(1.94) = 11.12$

$\mu + k\sigma = 15 + 2(1.94) = 18.88$

$P(X \leq 11.12 \text{ or } X \geq 18.88) = 0.041 + 0.024$

$= 0.065$

$k = 3$:

$\mu - k\sigma = 15 - 3(1.94) = 9.18$

$\mu + k\sigma = 15 + 3(1.94) = 20.82$

$P(X \leq 9.18 \text{ or } X \geq 20.82) = 0.004 + .004$

$= 0.008$

$K = 5$:

$\mu - K\sigma = 2.64 - 5(1.54) = -5.06$

$\mu + K\sigma = 2.64 + 5(1.54) = 10.34$

$P(X \leq -5.06 \text{ or } X \geq 10.34) \approx 0$

$K = 10$:

$\mu - K\sigma = 2.64 - 10(1.54) = -12.76$

$\mu + K\sigma = 2.64 + 10(1.54) = 18.04$

$P(X \leq -12.76 \text{ or } X \geq 18.04) \approx 0$

These values show that the bound $1/K^2$ is very conservative.

c)

80.) X = number of tornadoes observed in a 1-year period

$\lambda = 8$

a.- $P(X \leq 5) = F(5; 8) = .191$

b.- $P(6 \leq X \leq 9) = F(9; 8) - F(5; 8)$
$= 0.717 - 0.191 = 0.526$

c.- $P(10 \leq x) = 1 - P(X \leq 9) = 1 - 0.717 = 0.283$

d.- $E(X) = \lambda = 8$
$V(X) = \lambda = 8$
$\sigma(X) = \sqrt{8} \approx 2.83$
$P(X > 8 + 2.83 = 10.83) = P(X \geq 11)$
$= 1 - P(X \leq 10)$
$= 1 - 0.016$
$= 0.184$

108.) $n = 8 + 4 = 12$
$P(X = x) = h(x; n, M, N) = h(x; 4, 4, 12)$

$E(X) = n \cdot \frac{M}{N} = 4 \cdot \frac{4}{12} = \frac{16}{12} = 1.33 \downarrow$

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