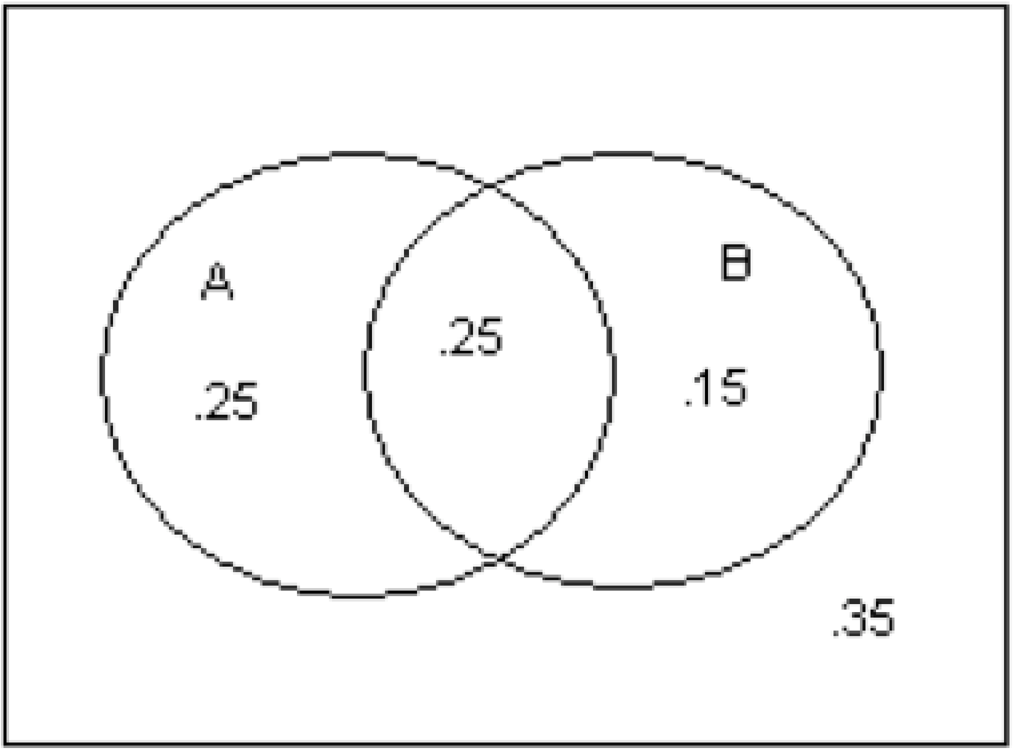
Statistics 1211 (4)
Professor Salzman
Homework#2
Solutions – Section 2

Chapter 2.4.47 (10 Points)



a. $P(B \mid A) = \dfrac{P(A \cap B)}{P(A)} = \dfrac{0.25}{0.50} = 0.50$

b. $P(B' \mid A) = \dfrac{P(A \cap B')}{P(A)} = \dfrac{0.25}{0.50} = 0.50$

c. $P(A \mid B) = \dfrac{P(A \cap B)}{P(B)} = \dfrac{0.25}{0.40} = 0.625$

d. $P(A' \mid B) = \dfrac{P(A' \cap B)}{P(B)} = \dfrac{0.15}{0.40} = 0.375$

e. $P(A \mid A \cup B) = \dfrac{P(A \cap (A \cup B))}{P(A \cup B)} = \dfrac{0.50}{0.65} = 0.7692$

Chapter 2.4.50 (10 Points)

a.

$P(M \cap LS \cap PR) = 0.05$ directly from the table of probabilities

b.

$P(M \cap \text{Pr}) = P(M, \text{Pr}, LS) + P(M, \text{Pr}, SS) = 0.05 + 0.07 = 0.12$

c.

$P(SS) = sum\ of\ 9\ probabilities\ in\ SS\ table = 0.56$

d.

$P(M) = 0.08 + 0.07 + 0.10 + 0.05 + 0.07 = 0.49$

$P(\mathrm{Pr}) = 0.02 + 0.07 + 0.07 + 0.02 + 0.05 + 0.02 = 0.25$

e.

$$P(M \mid SS \cap Pl) = \frac{P(M \cap SS \cap Pl)}{P(SS \cap Pl)} = \frac{0.08}{0.04 + 0.08 + 0.03} = 0.533$$

f.

$$P(SS \mid M \cap Pl) = \frac{P(SS \cap M \cap Pl)}{P(M \cap Pl)} = \frac{0.08}{0.08 + 0.10} = 0.44$$

$$P(LS \mid M \cap Pl) = 1 - P(SS \mid M \cap Pl) = 1 = 0.44 = 0.556$$

Chapter 2.4.61 (10 Points)

$P(0\ def\ in\ sample \mid 0\ def\ in\ batch) = 1$

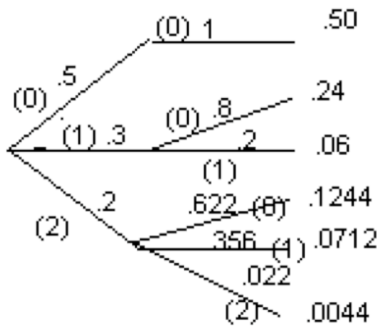
$$P(0\ def\ in\ sample \mid 1\ def\ in\ batch) = \frac{\binom{9}{2}}{\binom{10}{2}} = 0.800$$

$$P(1\ def\ in\ sample \mid 1\ def\ in\ batch) = \frac{\binom{9}{1}}{\binom{10}{2}} = 0.200$$

$$P(0\ def\ in\ sample \mid 2\ def\ in\ batch) = \frac{\binom{8}{2}}{\binom{10}{2}} = 0.622$$

$$P(1\ def\ in\ sample \mid 2\ def\ in\ batch) = \frac{\binom{2}{1}\binom{8}{1}}{\binom{10}{2}} = 0.356$$

$$P(2\ def\ in\ sample \mid 2\ def\ in\ batch) = \frac{1}{\binom{10}{2}} = 0.022$$



a.

$$P(0\ def\ in\ batch \mid 0\ def\ in\ sample) = \frac{0.5}{0.5 + 0.24 + 0.1244} = 0.578$$

$$P(1\ def\ in\ batch \mid 0\ def\ in\ sample) = \frac{0.24}{0.5 + 0.24 + 0.1244} = 0.278$$

$$P(2\ def\ in\ batch \mid 0\ def\ in\ sample) = \frac{0.1244}{0.5 + 0.24 + 0.1244} = 0.144$$

b.

$$P(0\ def\ in\ batch \mid 1\ def\ in\ sample) = 0$$

$$P(1\ def\ in\ batch \mid 1\ def\ in\ sample) = \frac{0.06}{0.06 + 0.0712} = 0.457$$

$$P(2\ def\ in\ batch \mid 1\ def\ in\ sample) = \frac{0.0712}{0.06 + 0.0712} = 0.543$$

Chapter 2.4.66 (10 Points)

Let E, C, and L be the events associated with e-mail, cell phones, and laptops, respectively. We are told that P(E)=40%, P(C)=30%, P(L)=25%, $P(E \cap C) = 23\%, P(E' \cap C' \cap L') = 51\%, P(E \mid L) = 88\%, and\ P(L \mid C) = 70\%.$

- **a.** P(C|E) = P(E∩C)/P(E) = .23/.40 = .575
- **b.** P(C|L) = P(C∩L)/P(L) = P(C)P(L|C)/P(L) = .30(.70)/.25 = .84
- **c.** P(C|E∩L) = P(C∩E∩L)/P(E∩L). For the denominator, P(E∩L) = P(L)P(E|L) = (.25)(.88) = .22. For the numerator, use P(E∪C∪L) = 1 – P(E′∩C′∩L′) = 1 – .51 = .49 and write
  P(E∪C∪L) = P(C) + P(E) + P(L) – P(E∩C) – P(C∩L) – P(E∩L) + P(C∩E∩L)
  → .49 = .30 + .40 + .25 – .23 – .30(.70) – .22 + P(C∩E∩L) → P(C∩E∩L) = .20.
  So, finally, P(C|E∩L) = .20/.22 = .9091

Chapter 2.5.76 (10 Points)

P(no error on any particular question) = .9, so P(no error on any of the 10 questions) =$(.9)^{10}$ = .3487. Then P(at least one error) = 1 – $(.9)^{10}$ = .6513. For **p** replacing .1, the two probabilities are $(1-\mathbf{p})^n$ and $1 - (1-\mathbf{p})^n$.

Chapter 2.5.82 (10 Points)

Event A: { (3,1)(3,2)(3,3)(3,4)(3,5)(3,6) }, P(A) = $\frac{1}{6}$;

Event B: { (1,4)(2,4)(3,4)(4,4)(5,4)(6,4) }, P(B) = $\frac{1}{6}$;

Event C: { (1,6)(2,5)(3,4)(4,3)(5,2)(6,1) }, P(C) = $\frac{1}{6}$;

Event A∩B: { (3,4) }; P(A∩B) = $\frac{1}{36}$;

Event A∩C: { (3,4) }; P(A∩C) = $\frac{1}{36}$;

Event B∩C: { (3,4) }; P(A∩C) = $\frac{1}{36}$;

Event A∩B∩C: { (3,4) }; P(A∩B∩C) = $\frac{1}{36}$;

P(A)·P(B)= $\frac{1}{6} \cdot \frac{1}{6} = \frac{1}{36}$ =P(A∩B)

P(A)·P(C)= $\frac{1}{6} \cdot \frac{1}{6} = \frac{1}{36}$ =P(A∩C)

P(B)·P(C)= $\frac{1}{6} \cdot \frac{1}{6} = \frac{1}{36}$ =P(B∩C)

The events are pairwise independent.

P(A)·P(B) ·P(C)= $\frac{1}{6} \cdot \frac{1}{6} \cdot \frac{1}{6} = \frac{1}{216} \neq \frac{1}{36}$ = P(A∩B∩C)

The events are not mutually independent