



15                   Key terms: ANCOVA, ANOVA, Bayesian statistics, hierarchical model,  
16                   variance components

## 17    **1 Introduction**

18    Analysis of variance (ANOVA) is widely used in scientific research for testing  
19    complicated multiple hypotheses. As presented originally in Fisher's seminal work  
20    (Fisher, 1925), ANOVA can be seen as the collection of the calculus of sum of  
21    squares and the associated models and significance tests. These tests and models  
22    have had a profound impact on ecological studies. ANOVA provides the  
23    computational framework for the design and analysis of ecological experiments  
24    (Underwood, 1997). As a data analysis tool, ANOVA is used in ecology for both  
25    confirmative and explorative studies. When used in a confirmative study, the  
26    randomized experimental design ensures that the resulting difference between  
27    treatments can be unambiguously attributed to the cause we are interested in  
28    testing. When used in explorative studies, the ANOVA framework reflects a basic  
29    scientific belief that correlation implies a causal relationship (Shipley, 2000). The  
30    simple steps of ANOVA computation, along with the associated significance test  
31    (the  $F$ -test), allow quick implementation and seemingly straightforward  
32    interpretation of the results.

33    Interpretation of ANOVA results can be problematic. Difficulties arise when the  
34    normality and independence of the response data are not met, when the  
35    experimental design is nested, when using an unbalanced design, or when missing  
36    cells are present. More importantly, ANOVA results are difficult to explain in  
37    ecological terms because significance test results are usually not scientifically very  
38    informative (Anderson et al., 2000). On one hand, when an experiment is proposed,

39 we almost always have reasons to believe that a treatment effect exists. Therefore,  
40 we want to know the strength of the effect a treatment has on the outcome rather  
41 than whether the treatment has an effect on the outcome. By using a significance  
42 test basing the inference on the assumption of no treatment effect, we emphasize the  
43 type I error rate (erroneously reject the null hypothesis of no treatment effect) often  
44 at the expense of statistical power, especially when multiple comparison is used. On  
45 the other hand, a nonexistent treatment effect can be shown to be statistically  
46 significant if one tries often enough (hence the article by Ioannidis, 2005).

47 From this practical perspective, we find the concept of variance components (Searle  
48 et al., 1992) especially useful. The relative sizes of the two variances indicate the  
49 effects of the factors of interest. When graphically presented, this partitioning of  
50 total variance into compartments is actually more informative than the results of a  
51 significance test. The ambiguity and difficulty of ANOVA can be alleviated by using  
52 a multilevel (or hierarchical) modeling approach for ANOVA proposed by Gelman  
53 (2005). His method can be summarized as the estimation of the variance  
54 components and treatment effects using a hierarchical regression. The results are  
55 often presented graphically. This approach is intuitively appealing and its  
56 implementation is straightforward even when the experimental design is nested and  
57 the response variable is not normally distributed. The multilevel ANOVA is  
58 Bayesian and inference about treatment effects are made using Bayesian posterior  
59 distributions of the parameters of interest. Gelman and Tuerlinckx (2000) suggested  
60 that the hierarchical Bayesian approach for ANOVA includes the classical ANOVA  
61 as a special case. We introduce Gelman's multilevel ANOVA using three examples.  
62 Statistical background are presented in Gelman (2005), Gelman and Hill (2007),  
63 and Gelman and Tuerlinckx (2000), and are briefly discussed in the supplementary  
64 materials.

## 2 Methods

We illustrate the multilevel ANOVA approach using a one-way ANOVA setting. For a one-way ANOVA problem, we have a treatment with several levels, and the statistical model is:

$$y_{ij} = \beta_0 + \beta_i + \epsilon_{ij}. \quad (1)$$

where  $\beta_0$  is the overall mean,  $\beta_i$  is the treatment effect for level  $i$  and  $\sum \beta_i = 0$ , and  $j$  represents individual observations in treatment  $i$ . This is a multilevel problem because we are interested in parameters at two levels: the data level and the treatment level. At the data level, individual response variable values are governed by the group level parameters, and the group level parameters are further governed by a distribution with hyper-parameters. The term “multilevel” is used to describe the data structure (data points are clustered in multiple treatment levels) and the hierarchical model structure. We avoid the usually used terms “fixed” or “random” effects to avoid confusion as described in Gelman and Hill (2007, sections 1.1 and 11.4) and Gelman (2005, section 6). The term “multilevel” encompasses both fixed and random effects. The total variance in  $y_{ij}$  is partitioned into between group variance ( $var(\beta_i)$ ) and within group variance ( $var(\epsilon_{ij})$ ). Instead of using the sum-of-squares calculation, we use a hierarchical formulation and model the coefficients  $\beta_i$  as a sample from a normal distribution with mean 0 and variance  $\sigma_\beta^2$ :

$$\beta_i \sim N(0, \sigma_\beta^2) \quad (2)$$

The model error term  $\epsilon_{ij}$  is also modeled as from a normal distribution:

$$\epsilon_{ij} \sim N(0, \sigma^2)$$

Or, equivalently,

$$y_{ij} \sim N(\beta_0 + \beta_i, \sigma^2) \quad (3)$$

89 The variance component for the treatment can be naturally estimated by  $\sigma_\beta$ , or the  
90 standard deviation of  $\hat{\beta}_i$  ( $s_\beta$ , the finite population standard deviation). The  
91 computation expressed here is standard for random effect coefficients under classical  
92 random effect model (Clayton, 1996). We can view fixed effects as special cases of  
93 random effects ( $\sigma_\beta = \infty$ ) in a Bayesian context. Therefore, this computation  
94 framework is not unique for Bayesian.

95 Model coefficients ( $\beta$ 's and  $\sigma_\beta$ , or  $s_\beta$  can be estimated using the maximum likelihood  
96 estimator, and the likelihood function of this setting is a product of two normal  
97 distribution density functions defined by equations 1 and 2. Analytical solutions are  
98 often available, but it is easy to implement the computation using Markov chain  
99 Monte Carlo simulation (MCMC, Gilks et al., 1997; Qian et al., 2003).

100 When there is more than one factor affecting the outcome, we can easily extend the  
101 approach by using the same hierarchical representation of the additional factors.

102 Furthermore, as suggested by equation 3, this approach is not limited by the  
103 normality assumption. That is, the normal distribution in equation 3 can be  
104 replaced with any distribution from the exponential family, similar to the  
105 generalization from linear models to the generalized linear models (GLM,  
106 McCullagh and Nelder, 1989).

## 107 2.1 Data Sets and Models

108 We illustrate the multilevel ANOVA using three data sets. The intertidal seaweed  
109 grazers example, a textbook example of ANOVA, is intended to make a direct  
110 comparison between the multilevel ANOVA and the classical ANOVA. The  
111 Liverpool moths example is used to illustrate the application of multilevel model  
112 under a logistic regression setting, where the response variable follows a binomial

distribution. The seedling recruitment data set illustrates the use of this approach for count data (Poisson regression) that may be spatially correlated. Results from applying the classical ANOVA or linear modeling are presented in the Supplementary Materials.

### 2.1.1 Intertidal Seaweed Grazers

This example was used in the text by Ramsey and Schafer (2002) (Case Study 13.1, p. 375), describing a randomized experiment designed to study the influence of three ocean grazers, small fish (f), large fish (F), and limpets (L), on regeneration rate of seaweed in the intertidal zone of the Oregon coast. The experiments were carried out in eight locations to cover a wide range of tidal conditions and six treatments were used to determine the effect of different grazers (C: control, no grazer allowed; L: only limpets are allowed; f: only small fish allowed; Lf: large fish excluded; fF: limpets excluded; and LfF: all allowed). The response variable is the seaweed recovery of the experimental plot, measured as percent of the plot covered by regenerated seaweed. The standard approach illustrated in Ramsey and Schafer (2002) is a two-way ANOVA (plus the interaction effect) on the logit transformed percent regeneration rates. The logit of percent regeneration rate is the logarithm of regeneration ratio (% regenerated over % not regenerated).

Using the multilevel notation, this two-way ANOVA model can be expressed as:

$$Y_{ijk} = \beta_0 + \beta_{1i} + \beta_{2j} + \beta_{3ij} + \epsilon_{ijk} \quad (4)$$

where  $Y$  is the logit of regeneration rate,  $\beta_{1i}$  is the treatment effect ( $i = 1, \dots, 6$  and  $\sum \beta_{1i} = 0$ ),  $\beta_{2j}$  is the block effect ( $j = 1, \dots, 8$  and  $\sum \beta_{2j} = 0$ ), and  $\beta_{3ij}$  is the interaction effect ( $\sum \beta_{3ij} = 0$ ). The residual term  $\epsilon_{ijk}$  is assumed to have a normal distribution with mean 0 and a constant variance, where  $k = 1, 2$  is the index of

individual observations within each Block – Treatment cell. The total variance in  $Y$  is partitioned into four components: treatment, block, interaction effects, and residual.

### 2.1.2 Liverpool Moths

Bishop (1972) reported a randomized experimental study on natural selection. The experiment was designed to answer the question that whether blackened tree trunks by air pollution near Liverpool, England were the cause of the increase of a dark morph of a local moth. The moths in question are nocturnal, resting during the day on tree trunks. In Liverpool, a high percentage of the moths are of a dark morph, whereas a higher percentage of the typical (pepper-and-salt) morph are observed in the Welsh countryside, where tree trunks are lighter. Bishop selected seven locations progressively farther away from Liverpool. At each location, Bishop chose eight trees at random. Equal numbers of dead light and dark moths were glued to the trunks in lifelike positions. After 24 hours, a count was taken of the numbers of each morph that had been removed – presumably by predators. The original study was published before the time of GLM, but the data set has since been used in several regression textbooks as an example of logistic regression (e.g., Ramsey and Schafer, 2002). We choose to model the moth data as from a binomial distribution and use the typical logistic regression model:

$$y_{ij} \sim \text{Bin}(p_{ij}, n_{ij}) \tag{5}$$

$$\text{logit}(p_{ij}) = \beta_0 + \beta_{1i} + \beta_{2i} \times \text{Dist}_j$$

where  $y_{ij}$  is the number of moths removed for morph  $i$  ( $i = 1$  (dark) or  $2$  (light)), at the  $j$ th distance (distance from Liverpool),  $n_{ij}$  is the total number of moths placed on each tree,  $p_{ij}$ , the parameter of interest, is the probability a moth being removed,  $\beta_{1i}$  is the morph effect, and  $\beta_{2i}$  is the slope on distance, representing the interaction

161 between morph color and distance. Bishop (1972) and Ramsey and Schafer (2002)  
162 used a categorical predictor “site” instead of distance from Liverpool to account for  
163 the apparent outlier at distance of 30.2 km (Figure 3 of the supplementary  
164 materials). We choose to use distance as a continuous predictor to illustrate the  
165 interaction effect between a categorical predictor and a continuous predictor.

### 166 2.1.3 Seedling Recruitment

167 Shen (2002) reported an observational study on factors affecting the species  
168 composition and diversity of a mixed evergreen-deciduous forest community in  
169 southwest China. The original study has observations at multiple spatial scales. We  
170 use the seedling recruitment data collected along a transect of 128  $5 \times 5$  meter  
171 consecutive plots to study factors affecting seedling recruitment. A total of 49  
172 species of seedlings were observed in the field and were classified into 5 types  
173 according to their status in community dynamics (Shen et al., 2000): Pioneer, Early  
174 dominant, Early companion, Later dominant (including evergreen species), and  
175 Tolerant. Within each plot, number of seedlings (height below 1 meter) was  
176 recorded, along with several physical and biological variables, including canopy gap  
177 measured in % (*Gap*), position of each plot measured as relative position along a  
178 hillside between valley (*Position* = 1) and ridge (*Position* = 5), soil total organic  
179 carbon (*TOC*, in %).

180 The observed number of seedlings (the response variable) from different plots are  
181 likely correlated, and the correlation is likely due to the spatial layout of the  
182 transect. A natural strategy for this problem is to introduce a spatial random  
183 effects term  $\epsilon$  using the intrinsic conditional autoregressive model (CAR) (Besag et

184 al., 1991; Qian et al., 2005):

$$185 \quad y_{ijkl} \sim Pois(\mu_{ij}) \quad (6)$$

$$\log(\mu_{ijk}) = \beta_0 + \beta_{1j} + \beta_{2k} + \beta_{3j} \times \text{logit}(Gap_i) + \beta_{4j} \times TOC + \epsilon_i + \varepsilon_i$$

186 where  $i$  is the index of plot,  $j$  is tree type index ( $j = 1, \dots, 5$ ), and  $k$  is the index of  
 187 position,  $l$  is the index of observations within a plot. The spatial random effect term  
 188  $\epsilon_i$  has a CAR prior. It is used to model the spatially structured variation. The error  
 189 term  $\varepsilon_i$  is used to account for unstructured over-dispersion. The sum of  $\epsilon_i$  and  $\varepsilon_i$  is  
 190 termed as the convolution prior (Besag et al., 1991). Only two-way interactions  
 191 between *Type* and the two continuous predictors were considered.

## 192 3 Results

193 Results from using classical approach are presented in the supplementary materials.  
 194 All multilevel results are presented graphically, showing the estimated posterior  
 195 mean (the circle), the 50% (thick line) and 95% (thin line) posterior credible  
 196 intervals. The ANOVA display shows the estimated posterior distributions of  
 197 variance components (in standard deviation), and the effects plots are based on  
 198 estimated posterior distributions of effects.

### 199 3.1 Seaweed Grazers

200 The multilevel model results are qualitatively similar to the conventional ANOVA  
 201 results (Figure 1). In addition, the estimated main effects are similar to results from  
 202 a conventional ANOVA (Figures 2 and 3). The emphasis on estimation is clearly  
 203 displayed in these plots. From the main effects plots, we know that the maximum  
 204 difference between treatment and control is about 3 in logit scale, or the mean  
 205 regeneration ratio of the control sites is about 20 times ( $e^3$ ) larger than that of the

206 treatment  $LfF$  sites. Traditional ANOVA does not emphasize the interaction effect  
207 beyond whether or not it is statistically significant. See supplementary materials for  
208 a comparison of the multilevel interaction plot and the commonly used interaction  
209 plot in ANOVA.

## 210 3.2 Moths

211 Using the multilevel model, we can use the traditional analysis of covariance  
212 (ANCOVA) setup to calculate the variance components of the main morph effect,  
213 main distance effect and the interaction (Figures 4-5). For this particular example,  
214 we see a strong interaction effect (clearly expressed by the difference in the  
215 distance slope in Figure 5) and an unambiguous morph main effect (Figure 4 right  
216 panel). The morph main effect is obvious because it is evaluated at a distance of  
217 27.2 (the average distance).

## 218 3.3 Seedling Recruitment

219 The largest variance component is the unstructured over-dispersion term, while the  
220 structured spatial random effects (CAR) contributes a smaller than expected  
221 variance (Figure 6). The inclusion of spatially correlated predictors may have  
222 accounted for some spatial autocorrelation in the response variable data. The  
223 variable *Type* explains the most variation in recruitment (Figures 6 and 7 left panel),  
224 which is expected since tree species were classified to reflect their different ecological  
225 strategies and roles in community dynamics. Similar to the GLM results, we found  
226 the effect of *GAP* is uncertain. However, the *Type* : *GAP* interaction effect, as  
227 shown in terms of type-specific *GAP* slope (Figure 8, right panel), indicates that  
228 type 4 (late dominant) trees are likely to respond negatively to increased gap, while

229 the rest likely respond positively. The interaction effect between *Type* and *TOC*  
230 (Figure 8, left panel) is unambiguous. Because soil carbon concentration is likely to  
231 be similar in neighboring plots, including a spatial autocorrelation term reduces  
232 uncertainty on type-specific *TOC* slopes. Type 4 and 5 trees include all evergreen  
233 species and the shade-tolerant deciduous species which tend to be restricted to  
234 relatively steep and higher hillside positions, corresponding to a lower soil *TOC*  
235 value; while the deciduous dominant and pioneer species normally achieve quick  
236 recruitment and fast growth in richer habitat. This pattern has also been reported  
237 in similar contexts (Tang and Ohsawa, 2002). Compared to the type-specific *TOC*  
238 slopes from the multilevel model (Figure 8, left panel), the GLM fit (supplementary  
239 materials) is quite different. Because spatial autocorrelation is accounted for, the  
240 multilevel model results are more reliable. The position main effect (Figure 7, right  
241 panel) shows a clear pattern indicating an increased recruitment as we move from  
242 valley to ridge. The large uncertainty associated with the position main effect can  
243 be attributed to the qualitative nature of this variable. That is, topographic relief  
244 has multiple scales while the plot size is fixed. Assigning position to a plot can be  
245 ambiguous depending on the length of a hillside. Plots with the same position level  
246 could be at quite different absolute positions on hillsides of different sizes. As a  
247 result, an emphasis on estimation is more informative than the hypothesis testing  
248 approach which will almost surely lead to non-significant result.

## 249 4 Discussion

250 Our examples used some typical data sets encountered in ecological studies.

251 Although ANOVA is well suited for analyzing the seaweed grazer data, multilevel  
252 ANOVA can be more informative and the graphical display is easier to understand

253 and interpret. In many ecological studies, data are collected from observations or  
254 from experiments applied to limited number of plots with unobserved confounding  
255 factors. Large natural variability plus small sample size often lead to non-significant  
256 results from ANOVA or  $t$ -tests, because the significance test is based on the  
257 comparison of the variance due to treatment and the residual variance. This  
258 situation is very common because of the high cost of collecting ecological data.  
259 When using the multilevel ANOVA, we estimate the treatment effect directly. The  
260 estimated treatment effect posterior distribution is not directly associated with the  
261 residual variance. As a result, we are more likely to show a significant treatment  
262 effect.

263 The seaweed regeneration example compares the multilevel ANOVA to the  
264 conventional ANOVA. The comparison illustrates the multilevel ANOVA's emphasis  
265 on estimation. This emphasis yields more informative results presented in terms of  
266 the estimated effects and the associated uncertainty. As we discussed in the  
267 supplementary materials, conventional hypothesis testing on treatment effects can  
268 be performed using the 95% posterior distributions of effects. As a result, our  
269 emphasis on estimation does not lead to lose of information in terms of comparisons  
270 of treatment effects. More importantly, the hierarchical computational framework  
271 allows ANOVA concept be applied to non-normal response variables, as illustrated  
272 in the Liverpool moth and seedling recruitment examples.

273 The philosophical basis of the traditional ANOVA is Popper's falsification theory  
274 (Popper, 1959). Although not fully compatible with methods practiced by most  
275 scientists, Popper's falsification philosophy had an immense impact on Fisher.

276 Because statistical theories are not strictly falsifiable, Fisher devised his  
277 methodology based on a quasi-falsificationist view. Fisher held that a statistical  
278 hypothesis should be rejected by any experimental evidence which, on the

279 assumption of that hypothesis, is relatively unlikely, relative that is to other possible  
280 outcomes of the experiment. Such tests, known as significance tests, or null  
281 hypothesis tests are controversial (see for example, Anderson et al., 2000 and Quinn  
282 and Keough, 2002).

283 Although Fisher's principles of randomized experimental design provide a  
284 mechanism for discerning the true causal effect of interest from confounding  
285 correlations, in practice ANOVA and associated significance tests are applied in  
286 both exploratory and confirmatory studies. In a confirmatory study, significance  
287 tests associated with ANOVA are used as the "seal of approval," while in an  
288 exploratory study ANOVA is often used to infer potential factors that may affect  
289 the outcome. While the null hypothesis of a significance test is of little interest, the  
290 variance component concept of ANOVA provides a convenient structure that allows  
291 scientists to form a causal model and develop hypotheses. The new computational  
292 method of multilevel ANOVA allows the classical ANOVA concept be applied to  
293 more complicated situations and can be accepted by both Bayesian and frequentist  
294 practitioners.

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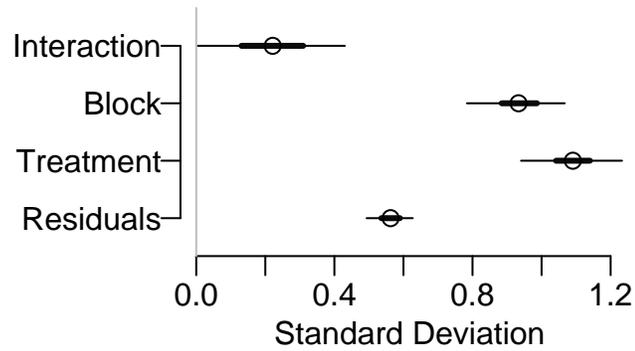


Figure 1: Seaweed Example: ANOVA display of the estimated standard deviation of the estimated variance components shows a similar general pattern as the conventional ANOVA results.

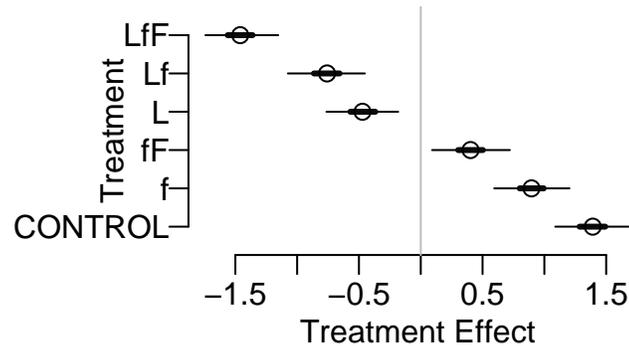


Figure 2: Estimated treatment main effect of the seaweed grazer example show that the regeneration rate decreases as grazing pressure increases. The largest difference between treatments is about 3 (in logit scale) or the regeneration ratio in CONTROL is about 20 times ( $e^3$ ) larger than the same in treatment  $LfF$ .

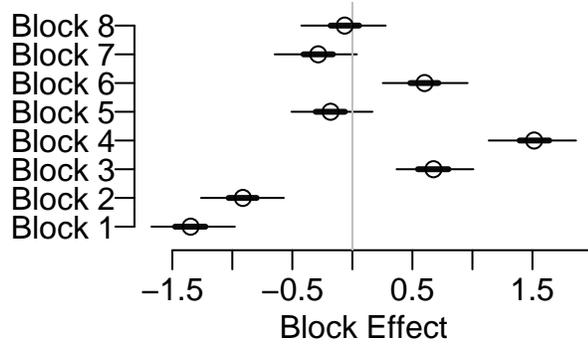


Figure 3: Estimated block main effect of the seaweed grazer example show the block effect has approximately the same magnitude as the treatment effect (3 in logit scale).

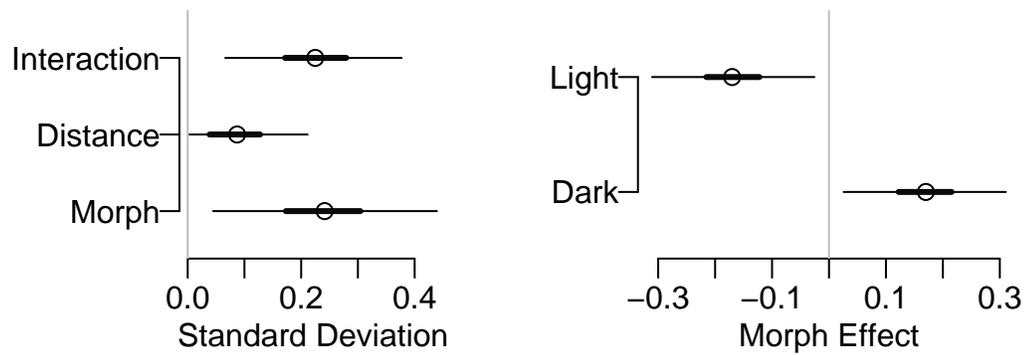


Figure 4: Liverpool moth Example: The left panel shows the ANOVA table indicating strong morph main effect and the morph-distance interaction effect. The right panel shows the estimated morph main effect.

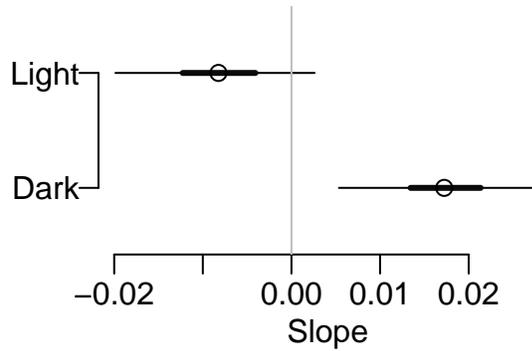


Figure 5: The estimated distance slope is positive for dark moths, indicating increased risk of removal for dark moths away from Liverpool. The distance slope for light moths is most likely negative, indicating increased risk of removal closer to Liverpool.

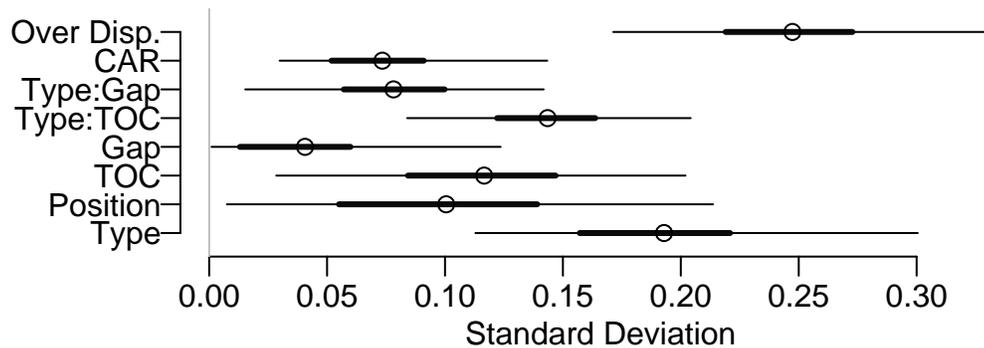


Figure 6: Seedling Example: ANOVA display of the estimated standard deviations of the estimated variance components shows that the unstructured overdispersion is the main contributor of the total variance, followed by tree type, type:TOC interaction, TOC, position, gap, and spatial autocorrelation (CAR).

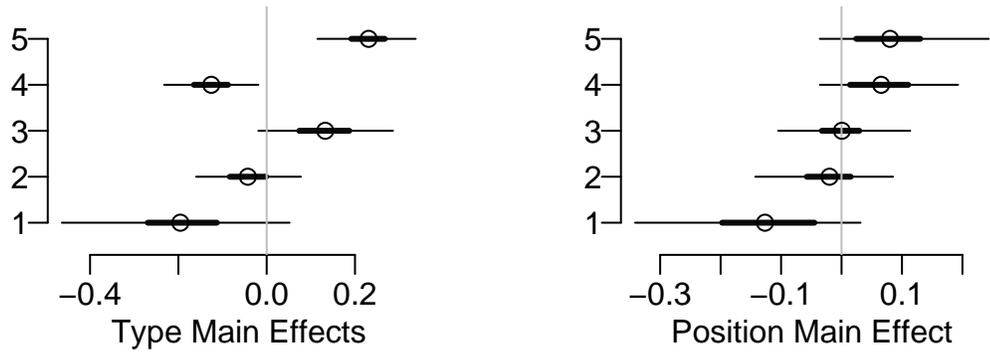


Figure 7: Seedling Example: The tree type main effect (left panel) shows that pioneer type (1) and late dominant (4) tend to have fewer seedlings and tolerant (5) tends to have much higher recruitment, while early dominant (2) and early companion (3) are close to average. The position main effect (right panel) shows that recruitment increases when moving from valley to ridge.

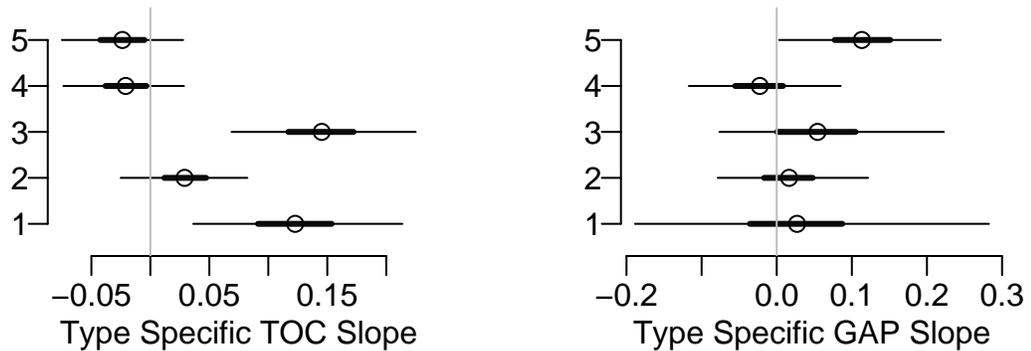


Figure 8: Seedling Example: The type:TOC interaction (left panel) shows positive generally slopes for pioneer (1), early dominant (2), and early companion (3) species and generally negative slopes for late dominant (4) and tolerant (5) species. The type:Gap interaction (right panel) shows only tolerant species respond positively to Gap.