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Contrarian deterministic effects on opinion dynamics: "the hung elections scenario"

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Abstract

We study the effects of contrarians on the dynamics of opinion forming using the 2-state Galam opinion dynamics model. In a single update step, groups of a given size are defined and all agents in each group adopt the state of the local majority. In the absence of contrarians, the dynamics is fast and leads to a total polarization always along the initial majority (for groups of odd sizes). The introduction of contrarians is then shown to give rise to interesting new dynamics properties. First, at low concentration a, a new mixed phase is stabilized with a coexistence of both states. This is an ordered phase with a clear cut majority–minority splitting (non zero order parameter). Second, there is a phase transition into a new disordered phase at $a_c = \frac{1}{6}$, 0.23, 0.30 $\ldots \frac{1}{2}$ for groups of respective sizes 3, 5, 9 and infinite. For $a \geqslant a_c$ the disordered phase has no opinion dominating with both state densities equal (zero order parameter). In this phase agents keep shifting states but no global symmetry breaking, i.e., the appearance of a majority, takes place. Our results may shed a new light on the phenomenon of "hung elections" as occured in the 2000 American presidential elections and that of the 2002 German parliamentary elections.

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In this paper, we study for the first time the effects of contrarian choices on the dynamics of opinion forming. A contrarian is an agent who adopts the choice opposite to the prevailing choice of others whatever this choice is [1]. The study of opinion forming using Statistical Physics has started more than 20 years ago from a founding

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work [2] which in addition to constructing a model for strike phenomena advocates the creation of the new field of Sociophysics. It is now becoming a very active field of research in Physics [3–11].

We start from the 2-state Galam opinion dynamics model where agents evolve by local majority rule updates. In a single step, groups of a given size are defined by picking up agents randomly. Then, in each group all agents adopt the state of the local majority. In the absence of contrarians, repeating the update process leads to a stable collective state with a total polarization of the opinion along either one of the two competing states A and B. The associated dynamics is fast and its direction is monitored by an unstable separator at some critical density p_c of agent supporting the A opinion.

In the case of odd size groups, $p_c = \frac{1}{2}$. By contrast, groups of even sizes make $p_c \neq \frac{1}{2}$. The dynamics of opinion for respectively A and B become assymmetric due to the possible occurrence of a tie. In the case of group of size 4 the unstable separator p_c is simultaneously at a value of 23% for one state and at 77% for the other [12,13]. Groups of large size accelerate the dynamics in reaching equilibrium. The corresponding number of required updates reduces drastically with increasing group size. It shrinks to one in the limit of one single grouping which includes the entire population.

Recently a generalization to a distribution of group sizes was achieved yielding a very rich and complex phase diagram [14]. The model was subsequently applied to rumor phenomena [15]. Studies by Krapivsky and Redner further explored the dynamical properties of the Galam model restricted to one group of size 3 [16].

Earlier version of this approach is found in the study of voting in hierarchical systems using real space renormalization group [13]. At each level of the hierarchy, groups of agents vote for a representative to a higher level using a local majority rule. In the mean field limit, going up the hierarchy turns out to be exactly identical to the above opinion forming process in terms of equations and dynamics. The probability of electing an A representative at some hierarchy level n is equal to the proportion of A opinions after n opinion updates [12,13].

Introduction of contrarians within above model restricted to groups of odd sizes is found to unfold the total polarization dynamics produced by the local majority rule at a low density a. The fully ordered state with one unique opinion becomes mixed with a stable majority–minority splitting. The symmetry breaking is preserved with a clear cut majority along the initial global majority and a nonzero-order parameter. The unstable separator is left unchanged at $p_c = \frac{1}{2}$.

However, at a critical concentration a_c, whose value depends on the group size, contrarians give rise to a critical behavior. We have $a_c = \frac{1}{6}$, 0.23, 0.30 $\ldots \frac{1}{2}$ for groups of respective sizes 3, 5, 9 and infinite. When $a > a_c$ a new disordered stable phase is obtained. In this phase there is no majority with neither opinion dominating the global opinion. Both state densities are equal with a zero order parameter. In this phase agents keep shifting states but no global symmetry breaking, i.e., the appearance of a majority, takes place. Above a_c contrarians have turned the unstable separator p_c into the unique stable attractor of the dynamics.

Our results may shed a new light on the phenomenon of "hung elections" as the ones which occurred in the 2000 American presidential elections and that of the 2002

German parliamentary elections. According to our model such events would not be chance driven but the deterministic outcome of increasing contrarian behavior. Such an hypothesis if it is proven true would imply the "hung elections scenario" to become a common occurrence in the future.

We start with the case of group size 3 [13,14]. At a given time t for a N person population we have $N_+(t)$ persons supporting A and $N_+(t)$ supporting B with $N_+(t) + N_-(t) = N$. These values can be evaluated using polls. Corresponding probabilities to an individual vote intention in favor of A or B writes,

$$p_\pm(t) \equiv \frac{N_\pm(t)}{N} , \tag{1}$$

with

$$p_+(t) + p_-(t) = 1 . \tag{2}$$

Accordingly, one cycle of local opinion updates via three persons grouping leads to a new distribution of vote intention as,

$$p_+(t+1) = p_+(t)^3 + 3p_+(t)^2 p_-(t) , \tag{3}$$

where $p_+(t+1) > p_+(t)$ if $p_+(t+1) > \frac{1}{2}$ and $p_+(t+1) < p_+(t)$ if $p_+(t+1) < \frac{1}{2}$. Indeed from Eq. (2) vote intention $p_+(t)$ flows monotonically toward either one of the two stable point attractors at $P_{+A} = 1$ and $P_{+B} = 0$. An unstable point separator attractor is located at $p_c = \frac{1}{2}$. It separates the two basins of attraction associated respectively to the point attractors.

During an election campaign people go through several successive different local discussions. To follow the associated vote intention evolution we iterate Eq. (2). A number of m discussion cycles gives the series $p_+(t+1)$, $p_+(t+2) \ldots p_+(t+m)$. For instance, starting at $p_+(t) = 0.45$ leads successively after five intention updates to the series $p_+(t+1) = 0.43$, $p_+(t+2) = 0.39$, $p_+(t+3) = 0.34$, $p_+(t+4) = 0.26$, $p_+(t+5) = 0.17$ with a continuous decline in A vote intentions. Adding three more cycles would result in zero. A vote intention with $p_+(t+6) = 0.08$, $p_+(t+7) = 0.02$ and $p_+(t+8) = 0.00$. Given any initial intention vote distribution, the random local opinion update leads toward a total polarization of the collective opinion. Individual and collective opinions stabilize simultaneously along the same and unique vote intention either A or B.

The update cycle number to reach either one of the two stable attractors can be evaluated from Eq. (2). It depends on the distance of the initial densities from the unstable point attractor. An analytic formula is derived below (see Eq. (6)). However, every update cycle takes some time length, which may correspond in real terms to some number of days. Therefore, in practical terms the required time to eventually complete the polarization process is much larger than the campaign duration, thus preventing it to occur. Accordingly, associate elections never take place at the stable attractors. From above example at $p_+(t) = 0.45$, two cycles yield a result of 39% in favor of A and 61% in favor of B. One additional update cycle makes 34% in favor of A and 66% in favor of B.

At this stage we are in a position to insert in the model the existence of contrarians. A contrarian is defined as follows. A contrarian arrives in a group like everyone with

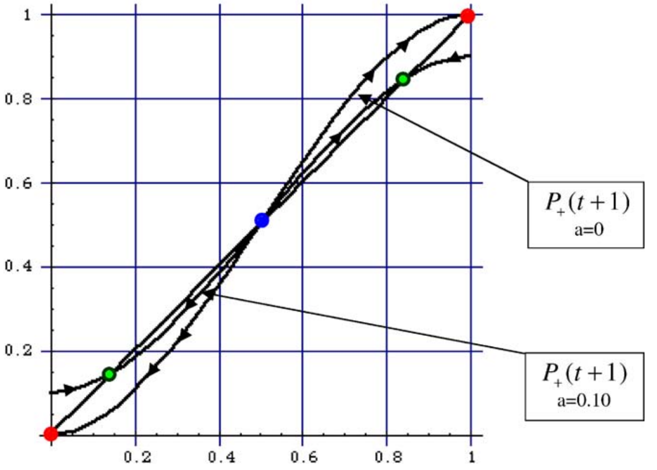

Fig. 1. Eq. (4) with $P_+(t+1)$ as function of $P_+(t)$ at respectively $a=0$ and $a=0.10$. In the second case the two stable point attractors have moved from total polarization towards coexistence of mixed vote intentions with a clear cut majority–minority splitting.

a fixed opinion which is used as others in the local update of the group. Then the local update takes place as before using a majority rule to select the new opinion shared by everyone in the group. However, once the contrarian leaves the group it immediately shifts to the opposite opinion. The shift is independent of the choice itself. Setting contrarian choices at a density a with $0 \leqslant a \leqslant 1$, the contrarian mechanism is defined as

$$\text{AAA and AAB} \rightarrow \text{AAA with probability } (1-a)^3 ,$$

$$\text{AAA and AAB} \rightarrow \text{AAB with probability } 3a(1-a)^2 ,$$

$$\text{AAA and AAB} \rightarrow \text{ABB with probability } 3a^2(1-a) ,$$

$$\text{AAA and AAB} \rightarrow \text{BBB with probability } a^3$$

and the symmetry with respect to A and B.

Combining above cases turns Eq. (2) for the density of A opinion into,

$$p_+(t+1) = (1-a)[p_+(t)^3 + 3p_+(t)^2 p_-(t)] + a[p_-(t)^3 + 3p_-(t)^2 p_+(t)] , \quad (4)$$

where first term corresponds to the regular update process and second term to contrarian contribution from local groups where the local majorities were in favor of B. From Eq. (4), the effect of low-density contrarian choices is readily seen as illustrated in Fig. 1 in the case $a = 0.10$, i.e., with 10% contrarian choices as compared to the pure case $a = 0$.

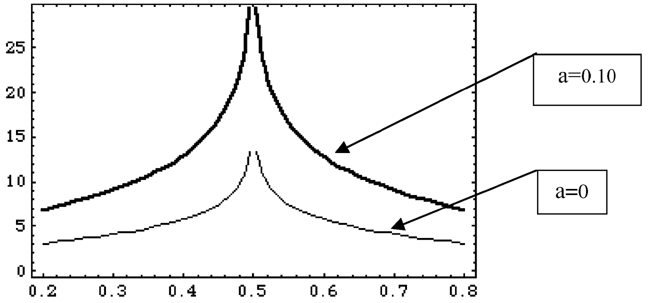

Fig. 2. Approximate number of cycles of vote intention updates to reach a total polarization of opinion as function of an initial support $P_+(t)$.

Main effects are two-fold. First both stable point attractors are shift toward coexistence vote intention values. Total polarization is averted with,

$$P_{+A(B)} = \frac{(2a-1) \pm \sqrt{12a^2 - 8a + 1}}{2(2a-1)}\ , \tag{5}$$

which are defined only in the range $a \leqslant \frac{1}{6}$. For instance, a value of $a = 0.10$ yields P_{+A}=0.85 and P_{+B}=0.15. At P_{+A}=0.85 exists a stable coexistence of vote intentions at respectively 0.85 in A's favor with 0.15 in B's favor. The reverse holds at $P_{+B} = 0.15$. At contrast contrarian choices keep unchanged the unstable point separator at $\frac{1}{2}$.

The second effect from contrarian choices is an increase in the number of cycle updates in reaching the stable attractors. For instance, starting as above at $p_+(t) = 0.45$ with a=0.10 leads now to the series $p_+(t+1)=0.44$, $p_+(t+2)=0.43$, $p_+(t+3)=0.42$, $p_+(t+4) = 0.40$, $p_+(t+5) = 0.38$. Additional 12 updates are required to reach the stable attractor at 0.15. All cycles score to 17 against only 8 without contrarian choices. A vote at two update cycles from same example would give a voting result of 43% in favor of A and 57% in favor of B, respectively, instead of 39% and 61% at $a = 0$.

An approximate formula can be derived from Eq. (4) to evaluate the update cycle number required to reach either one of the two stable attractors. It writes,

$$n \simeq \frac{1}{\ln\left[\frac{3}{2}(2a-1)\right]} \ln\left[\frac{p_c - P_S}{p_c - p_+(t)}\right] + \frac{1.85}{(2a-1)^{5.2}}\ , \tag{6}$$

where last term is a fitting correction. $P_S = P_{+B}$ if $p_+(t) < p_c$ while $P_S = P_{+A}$ when $p_+(t) > p_c$. The number of cycles being an integer, its value is obtained from Eq. (6) rounding to an integer. At $a = 0$, i.e., no contrarian choices, n is always a small number as shown in Fig. 2. Eq. (6) gives 8 at an initial value $p_+(t) = 0.45$ and 4 at $p_+(t) = 0.30$, which are the exact values obtained by successive iterations from Eq. (3). At $a = 0.10$ we found also the exact values of 17 and 9 as from Eq. (4).

Both Eq. (6) and Fig. 2 show explicitly the contrarian choice drastic effect in increasing the number of required levels to reach the stable point attractors. That means much longer real time. In practical terms it implies a quasi-stable coexistence of both

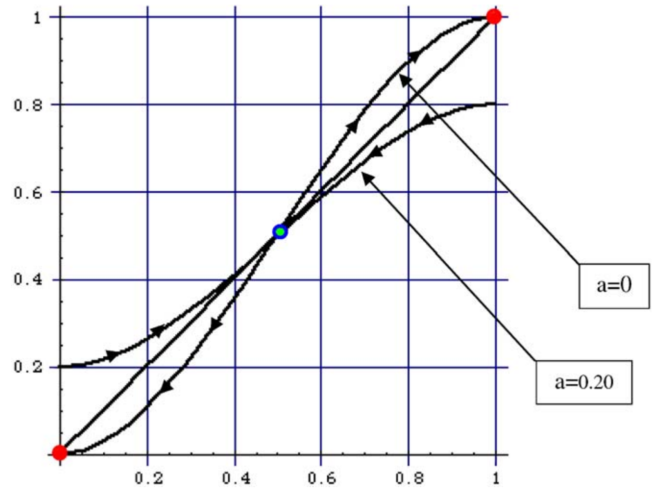

Fig. 3. $P_+(t+1)$ as function of $P_+(t)$ at $a=0$ and $a=0.20$. In the first case the vote intention flows away from the unstable point attractor at $\frac{1}{2}$ toward either one of the stable point attractors at zero or one. In the second case, contrarian choices have reversed the flow directions making any initial densities to flow toward $\frac{1}{2}$, the now stable and unique point attractor.

vote intentions not too far from fifty percent but yet with a clear-cut majority in one direction, which is determined by the initial majority.

However, contrarian choices may lead to a radical qualitative change in the whole vote intention dynamics. Eq. (5) shows that at a density of $a_c = \frac{1}{6} \simeq 0.17$, contrarian choices make both point attractors to merge simultaneously at the unstable point separator $p_c = \frac{1}{2}$ turning it to a stable point attractor. Consequences on the vote intention dynamics are drastic. The flow direction is reversed making any initial densities to converge toward a perfect equality between vote intention for A and B. In physical terms, contrarians produce a phase transition from a majority–minority phase into a fifty percent balance phase with no majority–minority splitting. In the ordered phase elections always yield a clear-cut majority. At contrast in the disordered phase elections lead to a random outcome driven by statistical fluctuations. An illustration is shown in Fig. 3 for 20% of contrarians.

In real social life people do not meet only by group of 3. However, generalizing above approach to larger sizes is straightforward and does not change the qualitative feature of the model. Dynamics reversal driven by contrarians towards the disorder phase with no majority–minority splitting is preserved. The main effect is an increase in the value of the contrarian critical density at which the phase transition occurs. In the case of an odd size k, Eq. (4) becomes

$$p_+(t+1) = (2a-1) \sum_{i=(k+1)/2}^{k} C_k^i \, p_+(t)^i \, p_-(t)^{(k-i)} + a \, , \tag{7}$$

where $C_k^i \equiv k!/(k-i)!i!$. The instrumental parameter in determining the flow direction and the associate phase transition is the eigenvalue at the point attractor $p_c = \frac{1}{2}$. It is given by

$$\lambda = (2a-1)\left[\frac{1}{2}\right]^{k-1} \sum_{i=(k+1)/2}^{k} (2i-k)C_k^i \ . \tag{8}$$

The range $\lambda > 1$ determines an unstable point attractor with an ordered phase characterized by the existence of a majority–minority splitting. At contrast, $\lambda < 1$ makes the point attractor stable. The case $\lambda = 1$ determines the critical value of the contrarian choice density a_c at which the phase transition occurs. From Eq. (8), we get,

$$a_c = \frac{1}{2}\left(1 - \left[\left(\frac{1}{2}\right)^{k-1} \sum_{i=(k+1)/2}^{k} (2i-k)C_k^i\right]^{-1}\right) \ . \tag{9}$$

In the case $k = 3$ we recover the above result $a = \frac{1}{6} \simeq 0.17$. From Eq. (9) it is seen that $a_c \to \frac{1}{2}$, $k \to +\infty$ with 0.23 at k=5 and 0.30 at $k = 9$.

We have presented a simple model to study the effect of contrarian choices on opinion forming. At low densities a the opinion dynamics leads to a mixed phase with a clear cut majority–minority splitting. However, beyond some critical density a_c, contrarians make all the attractors to merge at the separator p_c. It becomes the unique attractor of the opinion dynamics. When $a > a_c$ vote intentions flow deterministically with time towards an exact equality between A and B opinions. In this new disordered stable phase no majority appears. Agents keep shifting opinions but no symmetry breaking (i.e., the appearance of a majority) takes place. There an election would result in effect in a random winner due to statistical fluctuations. The value of a_c depends on the size distribution of update groups.

Our results may be put in parallel with recent "hung elections" in America (2000) and Germany (2002). It may suggest those were not chance driven, but instead the deterministic outcome of the existence of contrarians. Accordingly the associated "hanging chad elections" syndrome could become of a common occurrence in the near future.

While finalizing this manuscript we have notice Ref. [17] by Mobilia and Redner in which a phase transition in a disordered opinion phase is also obtained via another extension of Galam model (restricted to one group of size 3) which combines locally majority and minority rules. However the microscopic rules used as well as the socio-political interpretation and the critical values are different from those of the present work.

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