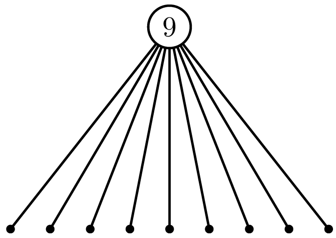
A. No Coalitions

A voter is decisive if the others are split 4-4:

$$\Pr(\text{Voter is decisive}) = \binom{8}{4} 2^{-8} = 0.273$$

Average $\Pr(\text{Voter is decisive}) = 0.273$



B. A Single Coalition of 5 Voters

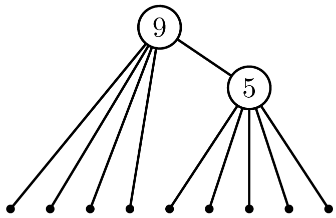
A voter in the coalition is decisive if others in the coalition are split 2-2:

$$\Pr(\text{Voter is decisive}) = \binom{4}{2} 2^{-4} = 0.375$$

A voter not in the coalition can never be decisive:

$$\Pr(\text{Voter is decisive}) = 0$$

Average $\Pr(\text{Voter is decisive}) = \frac{5}{9}(0.375) + \frac{4}{9}(0) = 0.208$



C. A Single Coalition of 3 Voters

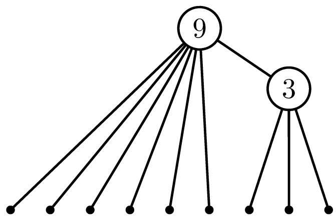
A voter in the coalition is decisive if others in the coalition are split 1-1 and the coalition is decisive:

$$\Pr(\text{Voter is decisive}) = \frac{1}{2} \cdot \frac{50}{64} = 0.391$$

A voter not in the coalition is decisive with probability:

$$\Pr(\text{Voter is decisive}) = \binom{5}{1} 2^{-5} = 0.156$$

Average $\Pr(\text{Voter is decisive}) = \frac{3}{9}(0.391) + \frac{6}{9}(0.156) = 0.234$



D. Three Coalitions of 3 Voters Each

A voter is decisive if others in the coalition are split 1-1 and the other two coalitions are split 1-1:

$$\Pr(\text{Voter is decisive}) = \frac{1}{2} \cdot \frac{1}{2} = 0.250$$

Average $\Pr(\text{Voter is decisive}) = 0.250$

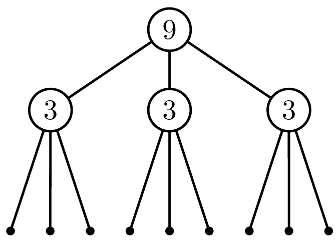


FIG. 1. *An example of four different systems of coalitions with nine voters, with the probability of decisiveness of each voter computed under the random voting model. Each is a "one person, one vote" system, but they have different implications for probabilities of casting a decisive vote. Joining a coalition is generally beneficial to those inside the coalition but hurts those outside. The average voting power is maximized under A, the popular-vote rule with no coalitions. From Katz, Gelman and King* (2002).