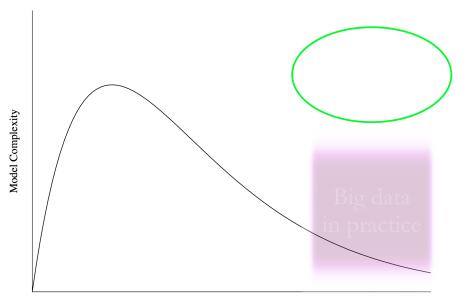
## Big Data need Big Model



## Stan: A platform for Bayesian inference

Andrew Gelman, Bob Carpenter, Matt Hoffman, Daniel Lee, Ben Goodrich, Michael Betancourt, Marcus Brubaker, Jiqiang Guo, Peter Li, Allen Riddell,...

Department of Statistics, Columbia University, New York (and other places)

10 Nov 2014





Web Images Videos Maps News More - Search tools

About 91,800,000 results (0.40 seconds)



Eminem - Stan (Short Version) ft. Dido - YouTube www.youtube.com/watch?v=aSLZFdqwh7E -

Artists: Eminem, Dido Album: No Angel

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Stan: A platform for Bayesian inference





#### Eminem - Stan (Short Version) ft. Dido - YouTube www.youtube.com/watch?v=aSLZFdgwh7E -

Artists: Eminem, Dido

Album: No Angel

Released: 1999

Feedback

#### Stan (song) - Wikipedia, the free encyclopedia en.wikipedia.org/wiki/Stan (song) -

"Stan" is the third single from the The Marshall Mathers LP, recorded in 1999 by American rapper Eminem and featuring British singer Dido. It peaked at number ... Thank You - Rock City - Robert Browning - Murder ballad

#### Stan: Project Home Page

#### mc-stan.org/ -

Stan modeling language and C++ library for Bayesian inference. NUTS adaptive HMC (MCMC) sampling, automatic differentiation, R, shell interfaces. Gelman.

#### Urban Dictionary: stan

#### www.urbandictionary.com/define.php?term=stan -

Based on the central character in the Eminem song of the same name, a "stan" is an overzealous maniacal fan for anv celebrity or athlete.

#### Gelman Carpenter Hoffman Lee Goodrich Betancourt ...

#### Stan: A platform for Bayesian inference



Stan: A platform for Bayesian inference

Stan is a probabilistic programming language implementing full Bayesian statistical inference with

• MCMC sampling (NUTS, HMC)

and penalized maximum likelihood estimation with

• Optimization (BFGS)

Stan is coded in C++ and runs on all major platforms (Linux, Mac, Windows).

Stan is freedom-respecting, open-source software (new BSD core, GPLv3 interfaces).

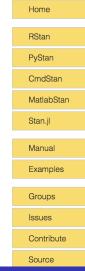
### Interfaces

Download and getting started instructions, organized by interface:

- RStan v2.5.0 (R)
- PyStan v2.5.0 (Python)
- CmdStan v2.5.0 (shell, command-line terminal)
- MatlabStan (MATLAB)
- Stan.jl (Julia)

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# Stan





## Ordered probit

```
data {
  int<lower=2> K:
  int<lower=0> N;
  int<lower=1> D:
  int<lower=1,upper=K> y[N];
  row_vector[D] x[N]; }
parameters {
  vector[D] beta;
  ordered[K-1] c; }
model {
  vector[K] theta;
  for (n in 1:N) {
    real eta;
    eta <- x[n] * beta;</pre>
    theta[1] <- 1 - Phi(eta - c[1]);
    for (k in 2:(K-1))
      theta[k] <- Phi(eta - c[k-1]) - Phi(eta - c[k]);
    theta[K] <- Phi(eta - c[K-1]);</pre>
    y[n] ~ categorical(theta);
  } }
                                                                         6/44
```

```
data {
  . . .
  real x_meas[N]; // measurement of x
  real<lower=0> tau; // measurement noise
}
parameters {
  real x[N];
                   // unknown true value
 real mu_x;
                   // prior location
  real sigma_x;
                   // prior scale
  . . .
}
model {
  x ~ normal(mu_x, sigma_x); // prior
  x_meas ~ normal(x, tau); // measurement model
  y ~ normal(alpha + beta * x, sigma);
  . . .
}
```



## Stan overview



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### Fit open-ended Bayesian models



- Fit open-ended Bayesian models
- Specify log posterior density in C++



- Fit open-ended Bayesian models
- ► Specify log posterior density in C++
- Code a distribution once, then use it everywhere



- Fit open-ended Bayesian models
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- Code a distribution once, then use it everywhere
- Hamiltonian No-U-Turn sampler



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- Autodiff



- Fit open-ended Bayesian models
- Specify log posterior density in C++
- Code a distribution once, then use it everywhere
- Hamiltonian No-U-Turn sampler
- Autodiff
- Runs from R, Python, Matlab, Julia; postprocessing



## People



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Stan core (15)



- Stan core (15)
- Research collaborators (30)
- Developers (100)



- Stan core (15)
- Research collaborators (30)
- Developers (100)
- User community (1000)



- Stan core (15)
- Research collaborators (30)
- Developers (100)
- User community (1000)
- Users (10000)



- National Science Foundation
- Institute for Education Sciences
- Department of Energy
- Novartis
- YouGov





 Bayesian inference for unsophisticated users (alternative to Stata, Bugs, etc.)



- Bayesian inference for unsophisticated users (alternative to Stata, Bugs, etc.)
- Bayesian inference for sophisticated users (alternative to programming it yourself)



- Bayesian inference for unsophisticated users (alternative to Stata, Bugs, etc.)
- Bayesian inference for sophisticated users (alternative to programming it yourself)
- Fast and scalable gradient computation



- Bayesian inference for unsophisticated users (alternative to Stata, Bugs, etc.)
- Bayesian inference for sophisticated users (alternative to programming it yourself)
- Fast and scalable gradient computation
- Environment for developing new algorithms



## ne social tv video games music apps

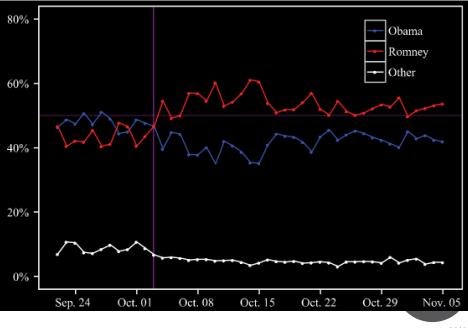


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Stan: A platform for Bayesian inference

# If the election were held today, who would you vote for?





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"This week, the New York Times and CBS News published a story using, in part, information from a non-probability, opt-in survey sparking concern among many in the polling community. In general, these methods have little grounding in theory and the results can vary widely based on the particular method used."

— Michael Link,

President, American Association for Public Opinion Research



**Michael W. Link** is Chief Methodologist for Research Methods at The Nielsen Company base of experience in survey research, having worked in academia (University of South (1999), not-for-profit research (RTI International, 1999-2004), government (Centers for Di Prevention, 2004-2007), and the private sector (Nielsen, 2007-present). He received his Science from the University of South Carolina. Michael's research centers around developmethodologies for confronting some of the most pressing issues facing survey research, techniques for improving survey participation and data quality (use of address-based san call screening technologies), methodological issues involving use of multiple modes in damail, CATI, field, mobile, meters), and obtaining participation from hard-to-survey popular isolated, racial and ethnic groups). His numerous research articles have appeared in *Pu Quarterly* and other leading scientific journals.

An AAPOR member since 1993, Michael served as AAPOR Conference Chair in back-to & 2010), a member of both the Cell Phone and Online task forces, an instructor for an AA numerous short-courses, a reviewer for the student paper competition on several occasion regular reviewer for *Public Opinion Quarterly*. He is a member of SAPOR, serving from 2 President, Conference Chair, and Student Paper Competition Organizer and also a mem

In 2011 he, along with several research colleagues, received AAPOR's Warren J. Mitofsl Award for their work on address based sampling designs. His current research focuses of technologies, such as mobile and social platforms, as vehicles for measuring and unders attitudes and behaviors. He will be teaching a short course on "The Role of New Technologies, or Replacing Traditional Surveys" at the 2012 AAPOR conference.

Gelman Carpenter Hoffman Lee Goodrich Betancourt ...

## Nielsen feels the heat of competition as it flubs its ratings of news broadcasts, putting ABC ahead of NBC



**BY DON KAPLAN** 

In spite of the goof, its global president took time to slam rival Rentrak, which collects different kind of data from viewers

NEW YORK DAILY NEWS / Sunday, October 19, 2014, 2:00 AM

AAA

#### MEDIA

# TV Ratings by Nielsen Had Errors for Months

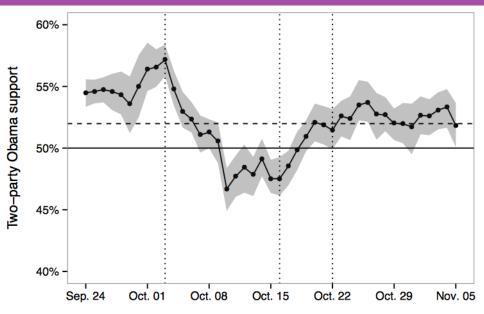
By BILL CARTER and EMILY STEEL OCT. 10, 2014

💙 Email

f Share

Nielsen, the television research firm, acknowledged on Friday that it had been reporting inaccurate ratings for the broadcast networks for the last seven months, a mistake that raises questions about the company's increasingly criticized system for measuring TV audiences.

## Xbox estimates, adjusting for demographics



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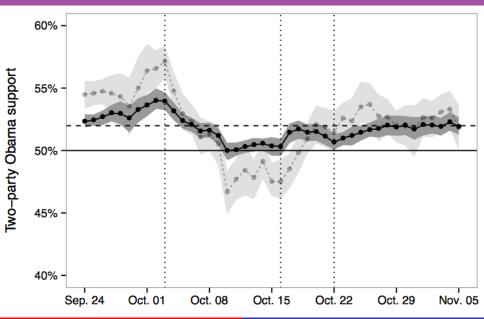
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- Karl Rove, Wall Street Journal, 7 Oct: "Mr. Romney's bounce is significant."
- Nate Silver, New York Times, 6 Oct: "Mr. Romney has only improved his own standing but also taken voters away the second standing but also taken voters away and the second standing but also taken voters away and the second standing but also taken voters away and the second standing but also taken voters away and the second standing but also taken voters away and the second standing but also taken voters away and the second standing but also taken voters away and the second standing but also taken voters away and taken voters away and the second standing but also taken voters away and the second standing but also taken voters away and the second standing but also taken voters away and the second standing but also taken voters away and the second standing but also taken voters away and taken voters awa only improved his own standing but also taken voters away from Mr. Obama's column."

## Xbox estimates, adjusting for demographics and partisanship



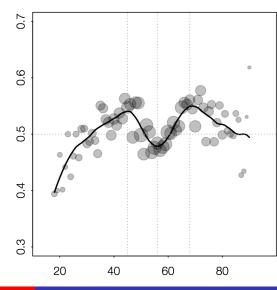
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## Jimmy Carter Republicans and George W. Bush Democrats

Republican Vote

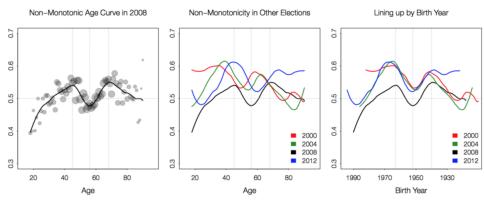


#### Non–Monotonic Age Curve in 2008



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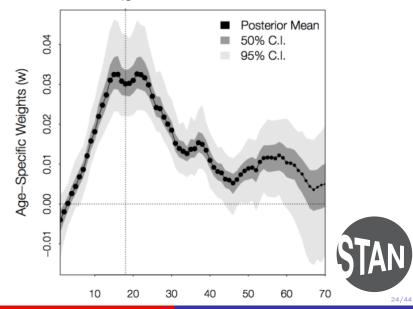
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#### The Formative Years

18



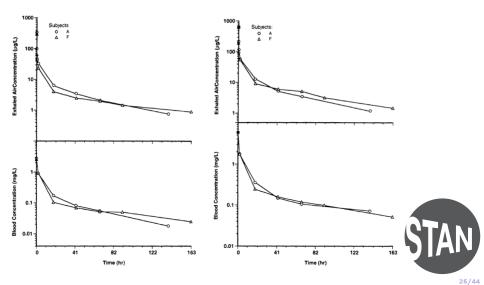
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# Toxicology

Exposure of 72 ppm

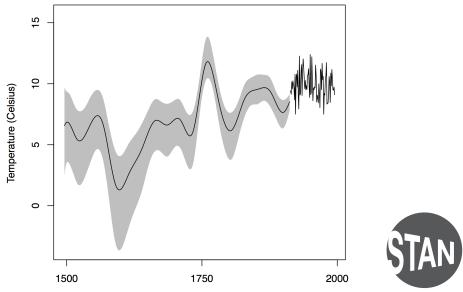
Exposure of 144 ppm



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#### Earth science



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#### Astronomy, ecology, linguistics, epidemiology, soil science, ...



#### Steps of Bayesian data analysis



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Model building



- Model building
- Inference



- Model building
- Inference
- Model checking



- Model building
- Inference
- Model checking
- Model understanding and improvement



#### Background on Bayesian computation



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Point estimates and standard errors



- Point estimates and standard errors
- Hierarchical models



- Point estimates and standard errors
- Hierarchical models
- Posterior simulation



- Point estimates and standard errors
- Hierarchical models
- Posterior simulation
- Markov chain Monte Carlo (Gibbs sampler and Metropolis algorithm)



- Point estimates and standard errors
- Hierarchical models
- Posterior simulation
- Markov chain Monte Carlo (Gibbs sampler and Metropolis algorithm)
- Hamiltonian Monte Carlo





> Problem: Gibbs too slow, Metropolis too problem-specific



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- > Problem: Gibbs too slow, Metropolis too problem-specific
- Solution: Hamiltonian Monte Carlo



- Problem: Gibbs too slow, Metropolis too problem-specific
- Solution: Hamiltonian Monte Carlo
- Problem: Interpreters too slow, won't scale



- Problem: Gibbs too slow, Metropolis too problem-specific
- Solution: Hamiltonian Monte Carlo
- Problem: Interpreters too slow, won't scale
- Solution: Compilation



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- ► Problem: Need gradients of log posterior for HMC



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- Problem: Need gradients of log posterior for HMC
- Solution: Reverse-mode algorithmic differentation



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- Problem: Existing algo-diff slow, limited, unextensible



- Problem: Gibbs too slow, Metropolis too problem-specific
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- ► Problem: Need gradients of log posterior for HMC
- Solution: Reverse-mode algorithmic differentation
- Problem: Existing algo-diff slow, limited, unextensible
- Solution: Our own algo-diff



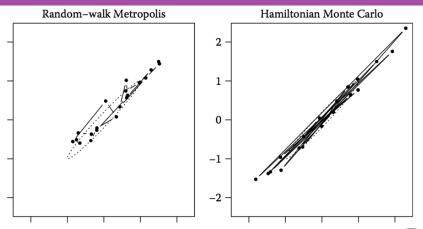
- Problem: Gibbs too slow, Metropolis too problem-specific
- Solution: Hamiltonian Monte Carlo
- Problem: Interpreters too slow, won't scale
- Solution: Compilation
- Problem: Need gradients of log posterior for HMC
- Solution: Reverse-mode algorithmic differentation
- Problem: Existing algo-diff slow, limited, unextensible
- Solution: Our own algo-diff
- Problem: Algo-diff requires fully templated functions



- > Problem: Gibbs too slow, Metropolis too problem-specific
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- Solution: Compilation
- Problem: Need gradients of log posterior for HMC
- Solution: Reverse-mode algorithmic differentation
- Problem: Existing algo-diff slow, limited, unextensible
- Solution: Our own algo-diff
- Problem: Algo-diff requires fully templated functions
- Solution: Our own density library, Eigen linear algebra



# Radford Neal (2011) on Hamiltonian Monte Carlo

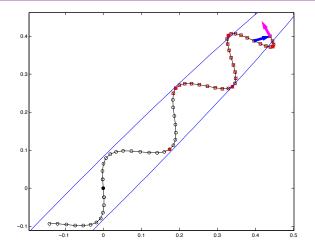


"One practical impediment to the use of Hamiltonian Monte Carlo is the need to select suitable values for the leapfrog stepsize,  $\epsilon$ , and the number of leapfrog steps L... Tuning HMC will usually require preliminary runs with trial values for  $\epsilon$  and L... Unfortunately, preliminary runs can be misleading ..."

Gelman Carpenter Hoffman Lee Goodrich Betancourt ...

- Created by Matt Hoffman
- Run the HMC steps until they start to turn around (bend with an angle > 180°)
- Computationally efficient
- Requires no tuning of #steps
- Complications to preserve detailed balance

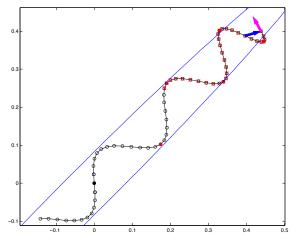






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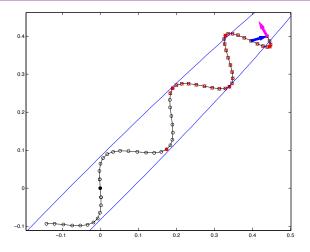


Blue ellipse is contour of target distribution



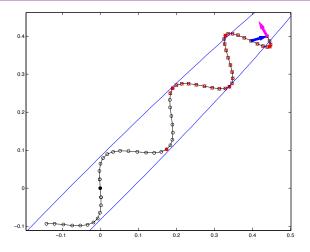
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- Blue ellipse is contour of target distribution
- Initial position at black solid circle



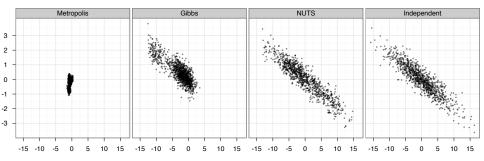


- Blue ellipse is contour of target distribution
- Initial position at black solid circle
- Arrows indicate a U-turn in momentum

Gelman Carpenter Hoffman Lee Goodrich Betancourt ...



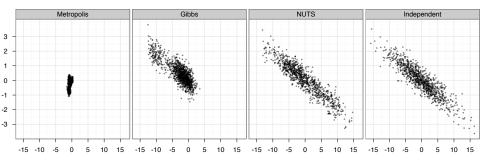
#### NUTS vs. Gibbs and Metropolis





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#### NUTS vs. Gibbs and Metropolis

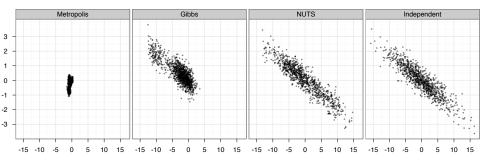


► Two dimensions of highly correlated 250-dim distribution



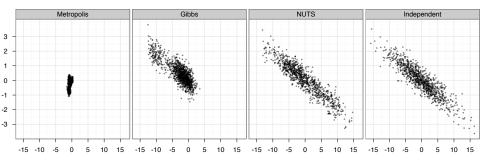
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#### NUTS vs. Gibbs and Metropolis



- Two dimensions of highly correlated 250-dim distribution
- IM samples from Metropolis, 1M from Gibbs (thin to 1K)

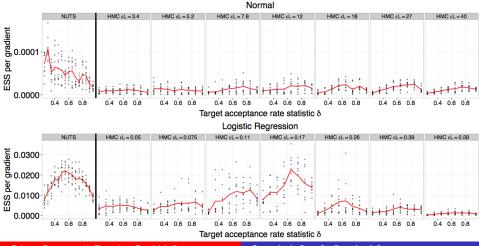
# NUTS vs. Gibbs and Metropolis



- Two dimensions of highly correlated 250-dim distribution
- 1M samples from Metropolis, 1M from Gibbs (thin to 1K)
- 1K samples from NUTS, 1K independent draws

# NUTS vs. Basic HMC

- > 250-D normal and logistic regression models
- Vertical axis shows effective #sims (big is good)
- (Left) NUTS; (Right) HMC with increasing  $t = \epsilon L$

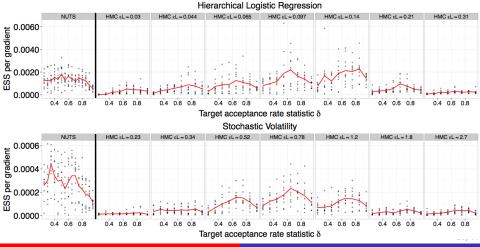


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# NUTS vs. Basic HMC II

- Hierarchical logistic regression and stochastic volatility
- Simulation time is step size e times #steps L
- NUTS can beat optimally tuned HMC



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Problem: Need ease of use of BUGS



- Problem: Need ease of use of BUGS
- Solution: Compile directed graphical model language



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- Problem: Need to tune parameters for HMC



- Problem: Need ease of use of BUGS
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- Solution: Auto tuning, adaptation



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- Solution: Density template metaprogramming



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- Problem: Limited error checking, recovery



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- Problem: Poor boundary behavior



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- ► Solution: Calculate limits (e.g.  $\lim_{x\to 0} x \log x$ )



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- ► *Problem*: Restrictive licensing (e.g., closed, GPL, etc.)



37/44

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- ► *Problem*: Restrictive licensing (e.g., closed, GPL, etc.)
- ► Solution: Open-source, BSD license



Simple harmonic oscillator:

$$\frac{dz_1}{dt} = -z_2$$
$$\frac{dz_2}{dt} = -z_1 - \theta z_2$$

with observations  $(y_1, y_2)_t, t = 1, \dots, T$ :

$$\begin{array}{rcl} y_{1t} & \sim & \mathsf{N}(z_1(t), \sigma_1^2) \\ y_{2t} & \sim & \mathsf{N}(z_2(t), \sigma_2^2) \end{array}$$

Given data  $(y_1, y_2)_t, t = 1, ..., T$ , estimate initial state  $(y_1, y_2)_{t=0}$  and parameter  $\theta$ 



## Stan program: 1

```
functions {
  real[] sho(real t, real[] y, real[] theta, real[] x_r, int[] x_i) {
    real dydt[2];
    dydt[1] <- y[2];</pre>
    dydt[2] <- -y[1] - theta[1] * y[2];</pre>
    return dydt;
  }
}
data {
  int<lower=1> T;
  real y[T,2];
  real t0;
  real ts[T];
}
transformed data {
  real x_r[0];
  int x_i[0];
}
```

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```
parameters {
  real y0[2];
  vector<lower=0>[2] sigma;
  real theta[1];
}
model {
  real z[T,2];
  sigma ~ cauchy(0,2.5);
  theta ~ normal(0,1);
  v0 \sim normal(0,1);
  z <- integrate_ode(sho, y0, t0, ts, theta, x_r, x_i);</pre>
  for (t in 1:T)
    y[t] ~ normal(z[t], sigma);
}
```



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### Stan output

Run RStan with data simulated from  $\theta = 0.15$ ,  $y_0 = (1, 0)$ , and  $\sigma = 0.1$ :

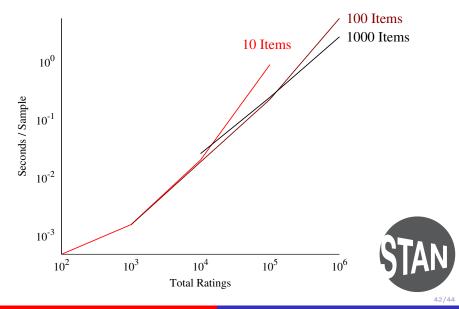
```
Inference for Stan model: sho.
4 chains, each with iter=2000; warmup=1000; thin=1;
post-warmup draws per chain=1000, total post-warmup draws=4000.
```

	mean	se_mean	sd	2.5%	25%	50%	75%	97.5%	n_eff	Rhat
y0[1]	1.05	0.00	0.09	0.87	0.98	1.05	1.10	1.23	1172	1
y0[2]	-0.06	0.00	0.06	-0.18	-0.10	-0.06	-0.02	0.06	1524	1
sigma[1]	0.14	0.00	0.04	0.08	0.11	0.13	0.16	0.25	1354	1
sigma[2]	0.11	0.00	0.03	0.06	0.08	0.10	0.12	0.18	1697	1
theta[1]	0.15	0.00	0.04	0.08	0.13	0.15	0.17	0.22	1112	1
lp	28.97	0.06	1.80	24.55	27.95	29.37	30.29	31.35	992	1



Gelman Carpenter Hoffman Lee Goodrich Betancourt ... Star

## Big Data, Big Model, Scalable Computing



Gelman Carpenter Hoffman Lee Goodrich Betancourt ... Stan: A platform

Stan: A platform for Bayesian inference

# Thinking about scalability



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#### Hierarchical item response model:

				St	an	JAGS	
# items	# raters	# groups	# data	time	memory	time	memory
20	2,000	100	40,000	:02m	16MB	:03m	220MB
40	8,000	200	320,000	:16m	92MB	:40m	1400MB
80	32,000	400	2,560,000	4h:10m	580MB	:??m	?MB



### Hierarchical item response model:

				St	an	JAGS		
# items	# raters	# groups	# data	time	memory	time	memory	
20	2,000	100	40,000	:02m	16MB	:03m	220MB	
40	8,000	200	320,000	:16m	92MB	:40m	1400MB	
80	32,000	400	2,560,000	4h:10m	580MB	:??m	?MB	

Also, Stan generated 4x effective sample size per iteration







Gelman Carpenter Hoffman Lee Goodrich Betancourt ... Stan: A platform for Bayesian inference

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Faster gradients and higher-order derivatives



- Faster gradients and higher-order derivatives
- Functions



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- Statistical algorithms



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  - ► (Penalized) mle



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  - ► (Penalized) mle
  - (Penalized) marginal mle



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  - (Penalized) mle
  - (Penalized) marginal mle
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  - (Penalized) marginal mle
  - Black-box variational Bayes
  - Data partitioning and expectation propagation

