Mathematical vs. statistical models in social science

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Themes

- Mathematical models in social science are cool ...
- But they tend to give qualitative rather than quantitative predictions
- Statistical modeling as an alternative
- Collaborations with Hayward Alker, Aaron Edlin, Noah Kaplan, Gary King, and Jonathan Katz
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Examples

- Political representation
- Trench warfare
- Rational voting
- Moderation and vote-getting
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Part 1: political representation
What does it mean to be “represented”?

- The U.S. is a representative democracy
- The right to vote; # representatives per voter
- Procedures vs. outcomes: what if 90% of the voters get the Congressmember whom they want?
- How close are actual elections?
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Congressional elections in 1948 and 1988

U.S. Congressional districts in 1948

U.S. Congressional districts in 1988

Democratic share of vote for Congress
Comparing to votes for President

Dem share of Congressional vote in 1988

Dem share of Congressional vote in 1948

Dem share of Presidential vote in 1988

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What does it mean to be “represented”? 

- Equal votes, satisfaction with outcomes, having your vote potentially matter
- Are your political views represented?
- Do your representatives look like you? Data from 1989?

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Seats and votes in a legislature

- Proportional representation in Europe
- No proportional representation in U.S.
- Wasted votes
- Small changes in votes
- Pinball analogy based on vote changes between election years
- No way to mathematically derive the “best” system
- Paradox of voting power and decisive votes
- Paradox of voting for native Australians
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- Small states overrepresented in U.S. Senate and electoral college
- Small states in U.S. get more than their share of gov’t funding
- Look at other countries: small states/provinces are generally overrepresented
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Part 2: trench warfare
Trench warfare: the live-and-let-live system

- Front-line troops in World War I avoided fighting (Ashworth book)
- Informal agreements across no-man’s-land
- How to understand this?
- Prisoner’s dilemma
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Prisoner’s dilemma for trench warfare

- Payoffs in the “game” (Axelrod book)
- No motivation to cooperate in single-play game
- Cooperation in repeated-play game
- Cool mathematical model
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Refuting the prisoner’s dilemma for trench warfare

- Look more carefully at payoffs
- No motivation to fight! Shooting poses a risk, whether or not the other side shoots
- Commanders manipulate the “game” to get soldiers to fight
- Hidden assumption of conventional roles of soldiers on opposing sides
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- “The evolution of cooperation”
- Using game theory to solve the “tragedy of the commons”
- Axelrod’s theory: politically liberal or conservative?
- “The norm of self-interest” (Miller)
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▶ How to defuse future conflicts?
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▶ Alternative strategy: set up immediate gains from cooperations and watch out for outside agents who could disrupt the cooperation
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Part 3: rational voting
Rational model for voting

- Utility of voting = $pB - c$:
  - $p =$ probability that a single vote will be decisive
  - $B =$ net benefit from your candidate winning
  - $c =$ net cost of voting (whether or not your candidate wins)

- Paradox of voting: $p$ is very small, so even for large values of $B$, there is no “instrumental” benefit to voting
- In presidential elections, $p$ is about 1 in 10 million
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- Utility of voting = $pB - c$
- “Benefit” of voting or “civic duty”
  - Does not explain higher turnout in close elections and more important elections
- Poor estimation of $p$
- Is voting irrational?
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Voting to benefit others

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  - $\alpha =$ (probably less than 1) discounts benefits to others
  - $N =$ number of persons affected by the election

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- Decoupling rationality from selfishness
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  - $N = \text{number of persons affected by the election}$
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- Decoupling *rationality* from *selfishness*
Example: a close election

- Each candidate expected to get between 47% and 53% of vote
  - Vote differential in range ±6%
  - $\Pr(\text{your vote is decisive}) \approx 1/(0.12n)$
- Suppose the selfish benefit to you is $10,000
- If $n = 1$ million, then expected selfish benefit is less than 10 cents
- Now consider a “social voter”

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Supporting evidence for the theory

- Small contributions to national campaigns
- Declining response rates in opinion polls
- Turnout is higher, not lower, in large elections
- Turnout is higher in close elections
- Strategic voting
- Voting on issues without direct instrumental benefits (abortion, All-Star game, Academy awards, . . .)
- Ask people why they vote
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- Are altruistic people more likely to vote?
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Part 4: candidate positioning
Candidate positioning

The “median voter theorem” (Hotelling, 1928):

![Diagram showing candidate positioning with a bell curve]

- **Left-wing**
- **D**
- **M**
- **R**
- **Right-wing**

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Author: Andrew Gelman

Title: Mathematical vs. statistical models in social science
Median voters and Newt Gingrich

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- The Democrats who lost were mostly moderate-to-conservative.
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Democrats running for reelection in 1994

Republicans running for reelection in 1994

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Congressmembers’ ideologies and median voters

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Estimating the electoral benefits of moderation

- Look at districts where Congressmembers are running for reelection
  - Predict their vote share given their "ideology score"
  - Also control for Presidential vote in previous election
- Noisy estimate in any particular year, so plot estimates over time
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Estimated effects of moderation for reelection vote

Analysis based on Poole’s dwnom1 score

<table>
<thead>
<tr>
<th>Year</th>
<th>Democratic Incumbents</th>
<th>Avg of Both Parties</th>
<th>Republican Incumbents</th>
</tr>
</thead>
<tbody>
<tr>
<td>1960</td>
<td>-2%</td>
<td>0%</td>
<td>-2%</td>
</tr>
<tr>
<td>1970</td>
<td>0%</td>
<td>2%</td>
<td>0%</td>
</tr>
<tr>
<td>1980</td>
<td>2%</td>
<td>4%</td>
<td>2%</td>
</tr>
<tr>
<td>2000</td>
<td>4%</td>
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Andrew Gelman

Mathematical vs. statistical models in social science
Return to the median voter theorem

- Is the median voter theorem “true”?
- No, and yes . . .
- Systematic differences between Democrats and Republicans, even in comparable districts
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  - Voting is irrational (unless you find it intrinsically enjoyable)
  - Politicians want to be at the median

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