

Computation for Bayesian Data Analysis

Andrew Gelman

10 August 2004

An example of Bayesian data analysis

- ▶ A problem in the study of social networks
- ▶ 3 models and Bayesian inference
- ▶ BUGS was too slow, so we used a program in R for Gibbs/Metropolis
- ▶ collaborators:
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 - ▶ Matt Salganik, Dept of Sociology, Columbia University
 - ▶ Jouni Kerman, Dept of Statistics, Columbia University
 - ▶ Peter Killworth and Chris McCarty shared their survey data

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Background: how many people do you know?

A model of overdispersion in social networks

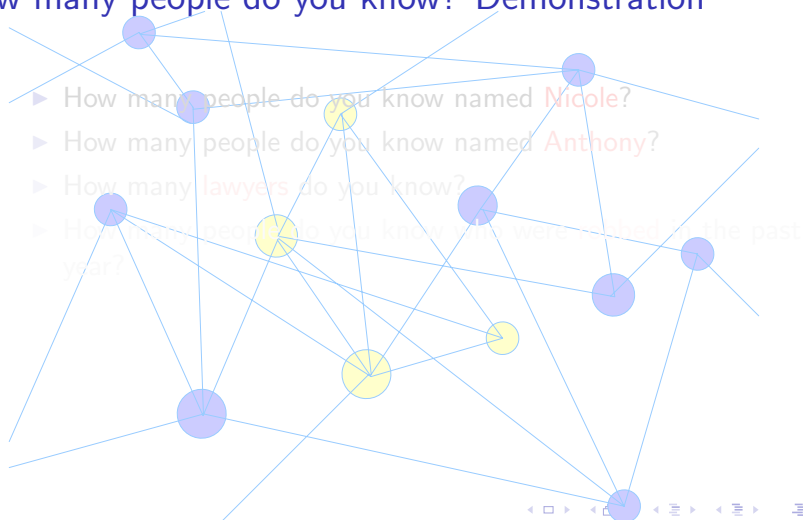
Fitting the model using the Gibbs/Metropolis sampler

Results from fitting the model

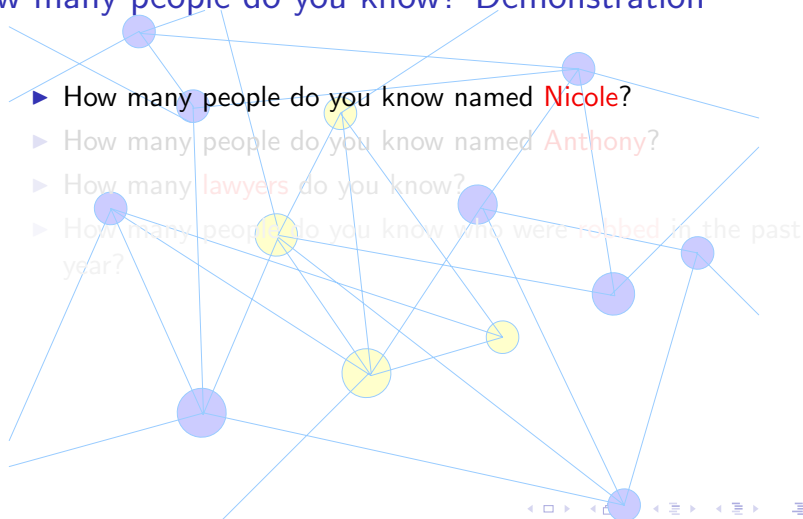
Confidence building

Summary

How many people do you know? Demonstration



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- ▶ How many people do you know named **Nicole**?
- ▶ How many people do you know named **Anthony**?
- ▶ How many **lawyers** do you know?
- ▶ How many people do you know who were **robbed** in the past year?



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Scale-up method: demonstration

- ▶ On average, you knew 0.6 Nicoles
- ▶ 0.13% of Americans are named Nicole
- ▶ Assume 0.13% of **your** acquaintances are Nicoles
- ▶ Estimate: on average, you know $0.6/0.0013 = 450$ people
- ▶ On average, you know 0.8 Anthonys
- ▶ 0.31% of Americans are named Anthony
- ▶ Estimate: on average, you know $1.6/0.0031 = 260$ people
- ▶ Why do these differ?

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Estimating group sizes: demonstration

- ▶ On average, you know 2.6 lawyers
 - ▶ Assume average network size is 450 people
 - ▶ Estimate: lawyers represent $2.6/450 = 0.58\%$ of the network
 - ▶ Estimate: $0.0058 \cdot 290 \text{ million} = 1.7 \text{ million lawyers in the U.S.}$
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- ▶ On average, you know 0.25 people who were robbed last year
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- ▶ Killworth, McCarty et al. surveys
- ▶ Scale-up method: how large is the average personal network?
- ▶ Estimating group sizes (e.g., American Indians)
- ▶ Can something be done with the 2-way data structure?

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Killworth, McCarty et al. surveys

- ▶ How many X's do you know?
- ▶ Stephanie, Jacqueline, Kimberly, Nicole, Christina, Jennifer
- ▶ Christopher, David, Anthony, Robert, James, Michael
- ▶ Twin, woman adopted kid in past year, gave birth in past year, widow(er) under 65
- ▶ Commercial pilot, gun dealer, postal worker, member of Jaycees, opened business in past year, American Indian
- ▶ Suicide in past year, died in auto accident, diabetic, kidney dialysis, AIDS, HIV-positive, rape victim, homicide victim, male in prison, homeless

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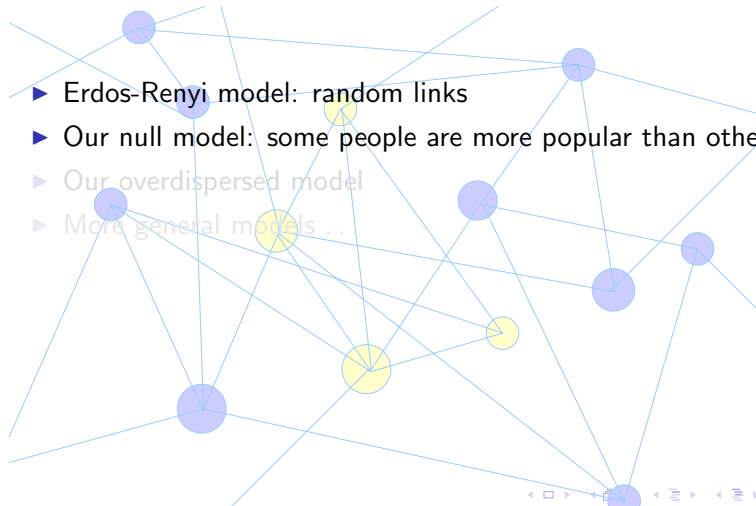
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Models of social network data

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- ▶ **Erdos-Renyi model: random links**
 - ▶ Our null model: some people are more popular than others
 - ▶ Our overdispersed model
 - ▶ More general models . . .

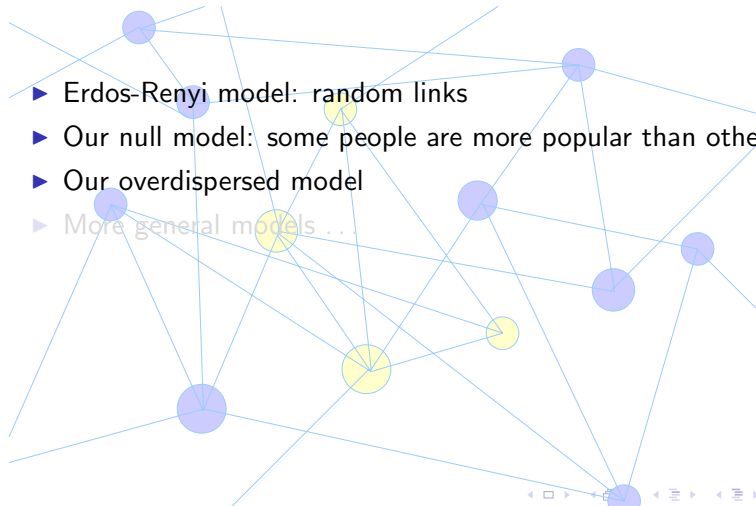
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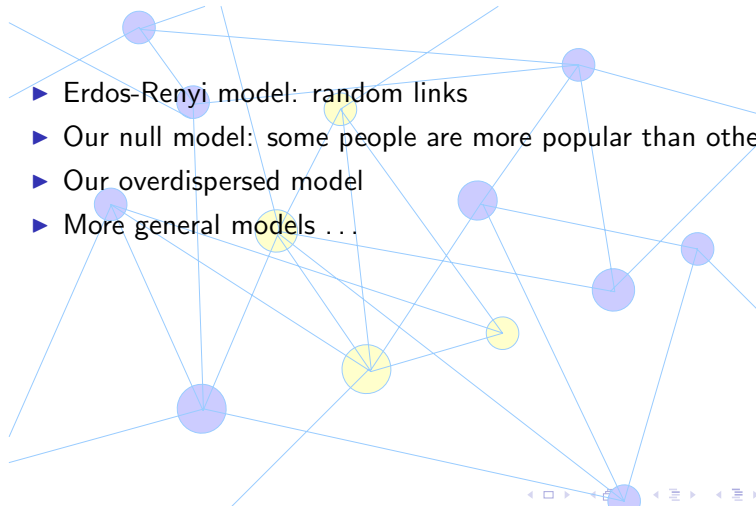
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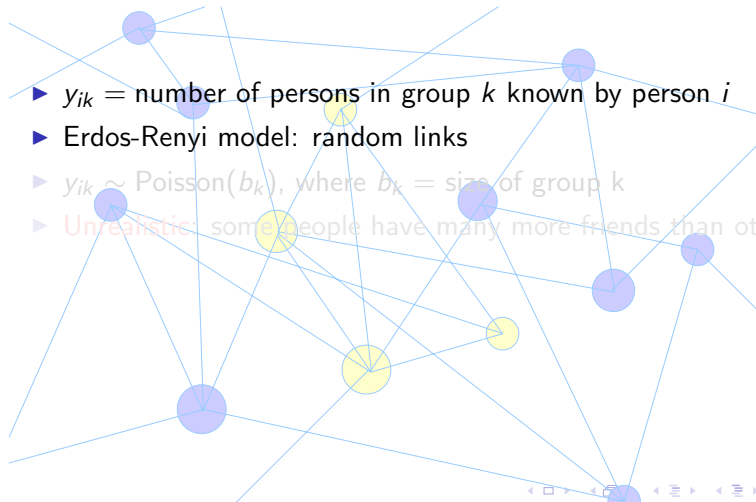
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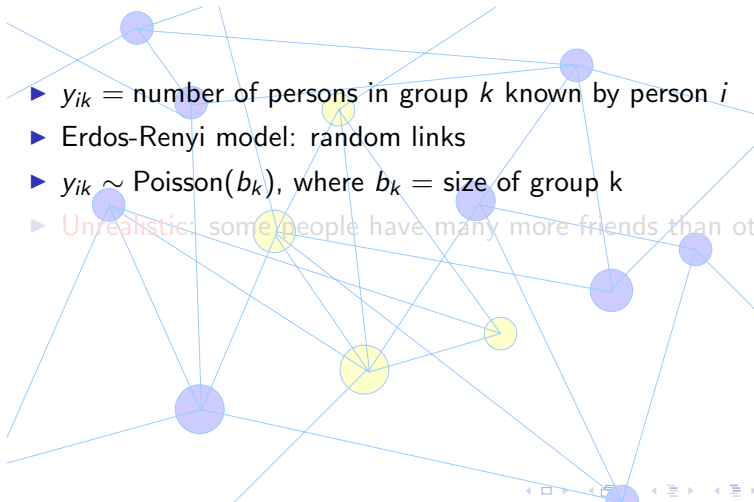
Erdos-Renyi model

- ▶ y_{ik} = number of persons in group k known by person i
- ▶ Erdos-Renyi model: random links
- ▶ $y_{ik} \sim \text{Poisson}(b_k)$, where b_k = size of group k
- ▶ **Unrealistic:** some people have many more friends than others



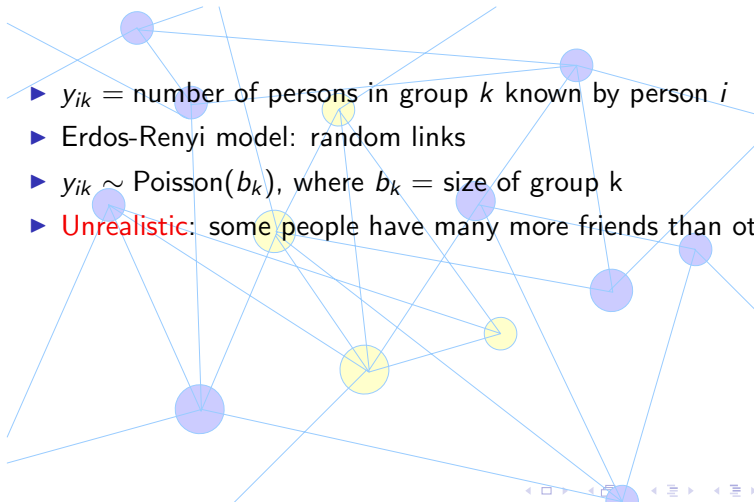
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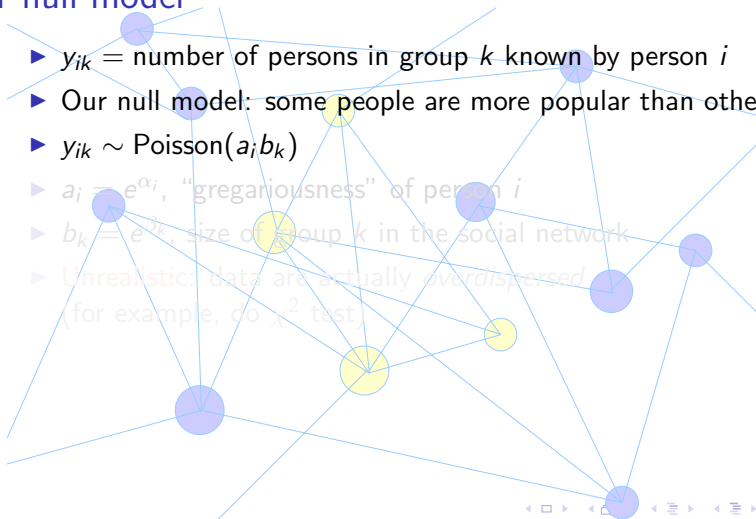


Our null model

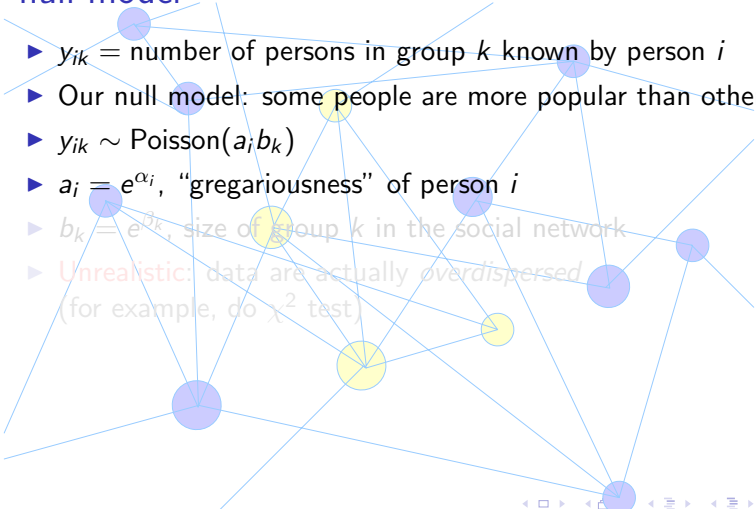
- ▶ y_{ik} = number of persons in group k known by person i
- ▶ Our null model: some people are more popular than others
- ▶ $y_{ik} \sim \text{Poisson}(a_i b_k)$
- ▶ $a_i = e^{\alpha_i}$, "gregariousness" of person i
- ▶ $b_k = e^{\beta_k}$, size of group k in the social network
- ▶ Unrealistic: data are actually overdispersed (for example, do χ^2 test)

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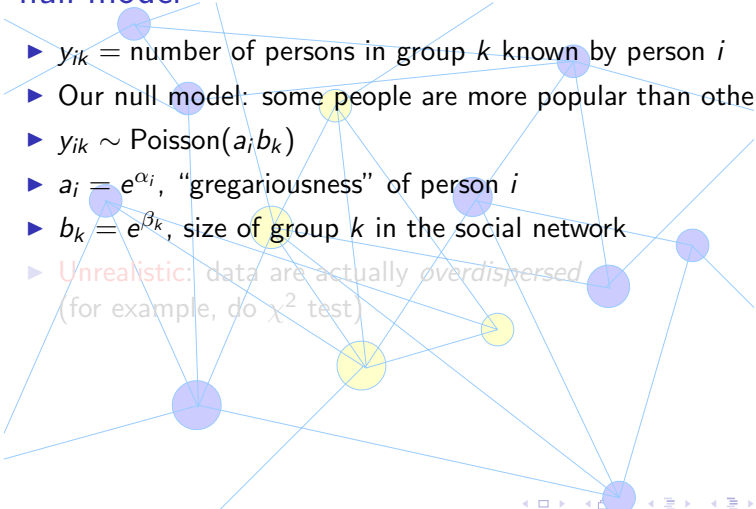
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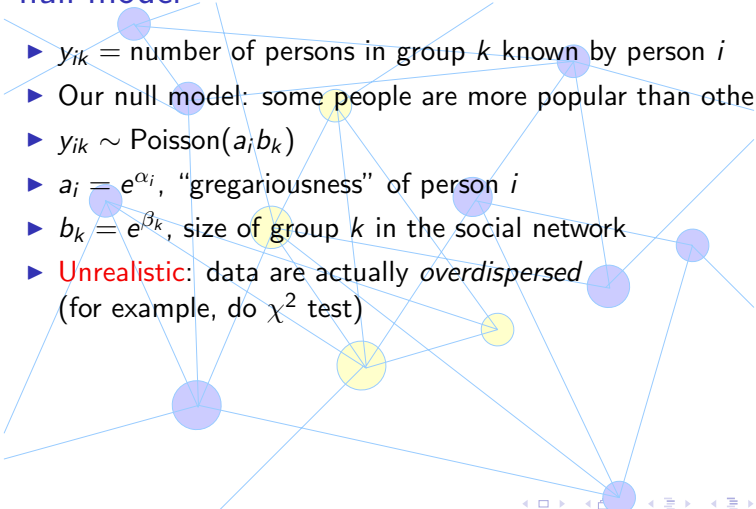
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Our overdispersed model

- ▶ y_{ik} = number of persons in group k known by person i
- ▶ Our overdispersed model: groups are *not* randomly spread in the population
- ▶ $y_{ik} \sim \text{Negative-binomial}(a_i, b_k, \omega_k)$
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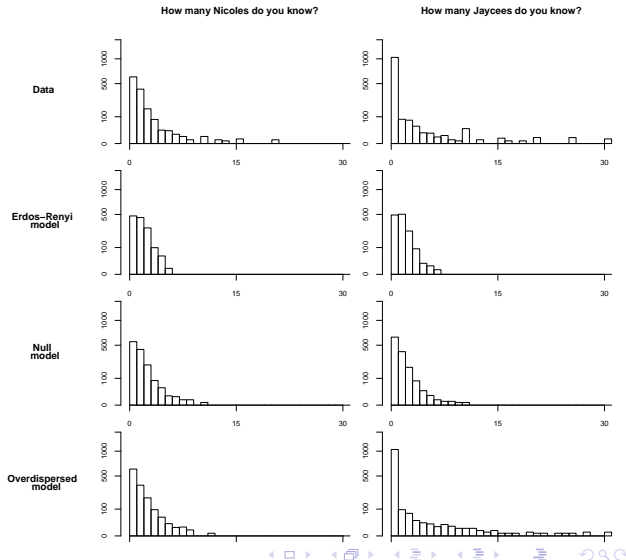
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Data, compared to simulations from 3 models



The overdispersed model

- ▶ data model: $y_{ik} \sim \text{Negative-binomial}(e^{\alpha_i + \beta_k}, \omega_k)$, for $i = 1, \dots, 1370$, $k = 1, \dots, 32$
- ▶ prior dists
 - ▶ $\alpha_i \sim N(\mu_\alpha, \sigma_\alpha^2)$, for $i = 1, \dots, 1370$
 - ▶ $\beta_k \sim N(\mu_\beta, \sigma_\beta^2)$, for $k = 1, \dots, 32$
 - ▶ $\omega_k \sim U(1, 20)$, for $k = 1, \dots, 32$
- ▶ hyperprior dist: $p(\mu_\alpha, \mu_\beta, \sigma_\alpha, \sigma_\beta) \propto 1$
- ▶ $1370 + 32 + 32 + 4$ parameters to estimate
- ▶ Nonidentifiability in $\alpha + \beta$ (to be discussed soon)

The overdispersed model

- ▶ data model: $y_{ik} \sim \text{Negative-binomial}(e^{\alpha_i + \beta_k}, \omega_k)$, for $i = 1, \dots, 1370$, $k = 1, \dots, 32$
- ▶ prior dists
 - ▶ $\alpha_i \sim N(\mu_\alpha, \sigma_\alpha^2)$, for $i = 1, \dots, 1370$
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 - ▶ $\omega_k \sim U(1, 20)$, for $k = 1, \dots, 32$
- ▶ hyperprior dist: $p(\mu_\alpha, \mu_\beta, \sigma_\alpha, \sigma_\beta) \propto 1$
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Latent-data parameterization

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- ▶ alternative using latent data γ_{ik} :
 - ▶ $y_{ik} \sim \text{Poisson}(e^{\alpha_i + \beta_k + \gamma_{ik}})$
 - ▶ $\gamma_{ik} \sim \text{log-gamma}(\text{shape parameter of } 1/(\omega_k - 1))$
- ▶ Not so helpful here, but this “data augmentation” idea is useful in other settings

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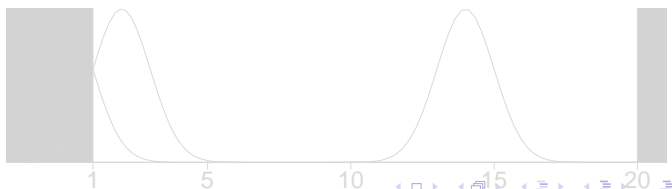
Gibbs-Metropolis algorithm: updating α, β, ω

- For each i , update α_i using Metropolis with jumping dist.
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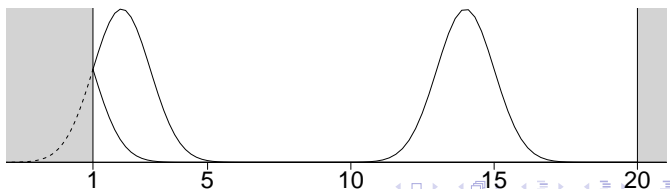
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Gibbs-Metropolis algorithm: updating hyperparameters

- ▶ Update $\mu_\alpha \sim N\left(\frac{1}{n}\sum_{i=1}^n \alpha_i, \frac{1}{n}\sigma^2\right)$
- ▶ Update $\sigma_\alpha^2 \sim \text{Inv-}\chi^2\left(n-1, \frac{1}{n}\sum_{i=1}^n (\alpha_i - \mu_\alpha)^2\right)$
- ▶ Similarly with μ_β, σ_β
- ▶ Renormalize to identify the α 's and β 's ...

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Renormalizing

- ▶ Problem: α_i 's and β_k 's are not separately identified in the model, $y_{ik} \sim \text{Negative-binomial}(e^{\alpha_i + \beta_k}, \omega_k)$
- ▶ Possible solutions:
 - ▶ Choose a "baseline" value: set $\alpha_1 = 0$ (for example)
 - ▶ Renormalize a group of parameters: set $\sum_{i=1}^n \alpha_i = 0$
 - ▶ Assume the prior distribution: $\alpha_i \sim \text{Normal}(\mu, \sigma^2)$
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Adaptive Metropolis jumping

- ▶ Parallel scalar updating of the components of α, β, ω
- ▶ Adapt each of $1370 + 32 + 32$ jumping scales to have $E(p_{\text{jump}}) \approx 0.44$
- ▶ Save p_{jump} from each Metropolis step, then average them and rescale every 50 iterations:
 - ▶ Where $\text{avg } p_{\text{jump}} > 0.44$, **increase** the jump scale
 - ▶ Where $\text{avg } p_{\text{jump}} < 0.44$, **decrease** the jump scale
- ▶ After burn-in, stop adapting
- ▶ If we had vector jumps, we would adapt the scale so that $E(p_{\text{jump}}) \approx 0.23$
- ▶ More effective adaptation uses avg. squared jumped distance

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Mixing (convergence) of simulations

- ▶ Simulate 3 parallel sequences, starting with draws from the prior distribution
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- ▶ Run 2000 iterations, discard first half: mixed ($\hat{R} \leq 1.1$ for all parameters)
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Computation in R

- ▶ BUGS was too slow (over 1400 parameters)
- ▶ Programming from scratch in R is awkward, buggy
- ▶ Instead, we use our general Gibbs/Metropolis programming environment
- ▶ Set up MCMC object
- ▶ Specify Gibbs updates
- ▶ Log-posterior density for Metropolis steps
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network.1 <- mcmcEngine (network.data, network.init,  
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network.update <- list(  
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    jump=Jump("omega.jump", lower=1.01, upper=20)),  
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network.1 <- mcmcEngine (network.data, network.init,  
  update=network.update, n.iter=1000, n.chains=3)  
network.update <- list(  
  alpha = Metropolis (f.logpost.alpha),  
  beta = Metropolis (f.logpost.beta),  
  omega = Metropolis (f.logpost.omega,  
    jump=Jump("omega.jump", lower=1.01, upper=20)),  
  mu.alpha = Gibbs (mu.alpha.update),  
  mu.beta = Gibbs (mu.beta.update),  
  sigma.alpha = Gibbs (sigma.alpha.update),  
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```

Data and initial values

```
y <- as.matrix (read.dta ("social.dta"))  
y <- y[1:50,]  
network.data <- list (y=y, data.n=nrow(y),  
  data.j=ncol(y))  
network.init <- function(){  
  alpha <- rnorm(data.n)  
  beta <- rnorm(data.j)  
  omega <- runif(data.j,1.01,20)  
  mu.alpha <- rnorm(1)  
  mu.beta <- rnorm(1)  
  sigma.alpha <- runif(1)  
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```

Gibbs samplers for the hyperparameters

```
mu.alpha.update <- function()  
  rnorm (1, mean(alpha), sigma.alpha/sqrt(data.n))  
mu.beta.update <- function()  
  rnorm (1, mean(beta), sigma.beta/sqrt(data.j))  
sigma.alpha.update <- function()  
  sqrt (sum((alpha-mu.alpha)^2)/rchisq(1, data.n-1))  
sigma.beta.update <- function()  
  sqrt (sum((beta-mu.beta)^2)/rchisq(1, data.j-1))
```

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```

Log-likelihood for each data point

```
f.loglik <- function (y, alpha, beta, omega, data.n){  
  theta.mat <- exp(outer(alpha, beta, "+"))  
  omega.mat <- outer(rep(0, data.n), omega, "+")  
  dnbinom (y, theta.mat/(omega.mat-1), 1/omega.mat,  
    log=T)}
```

Log-posterior density for each vector parameter

```
f.logpost.alpha <- function() {  
  loglik <- f.loglik (y, alpha, beta, omega, data.n)  
  rowSums (loglik, na.rm=TRUE) +  
    dnorm (alpha, mu.alpha, sigma.alpha, log=TRUE)}  
f.logpost.beta <- function() {  
  loglik <- f.loglik (y, alpha, beta, omega, data.n)  
  colSums (loglik, na.rm=TRUE) +  
    dnorm (beta, mu.beta, sigma.beta, log=TRUE)}  
f.logpost.omega <- function() {  
  loglik <- f.loglik (y, alpha, beta, omega, data.n)  
  colSums (loglik, na.rm=T)}
```

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```

Bounded jumping for the ω_k 's

Customized Metropolis jumping rule for the components of ω :

```
omega.jump <- function (omega, sigma) {  
  reflect (rnorm (length(omega), omega, sigma),  
    .lower, .upper)}
```

Renormalization of the α_i 's and β_k 's

```
renorm.network <- function() {  
  const <- log (sum(exp(beta[1:12]))/0.069)  
  alpha <- alpha + const  
  mu.alpha <- mu.alpha + const  
  beta <- beta - const  
  mu.beta <- mu.beta - const}
```


Running MCMC and looking at the output

```
net <- run(network.1)
attach (as.rv (net))
```

Some output:

name	mean	sd	25%	50%	75%	Rhat
beta[1]	-5.1	0.1	(-5.4	-5.2	-5.1)	1.0
beta[2]	-6.4	0.1	(-6.9	-6.7	-6.5)	1.2
beta[3]	-6.1	0.1	(-6.5	-6.3	-6.2)	1.1
beta[4]	-7.0	0.2	(-7.6	-7.4	-7.1)	1.0
beta[5]	-5.1	0.1	(-5.4	-5.3	-5.2)	1.2
beta[6]	-5.6	0.2	(-6.1	-5.9	-5.8)	1.0

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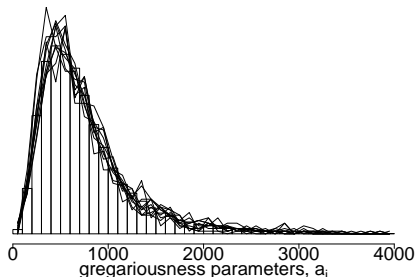
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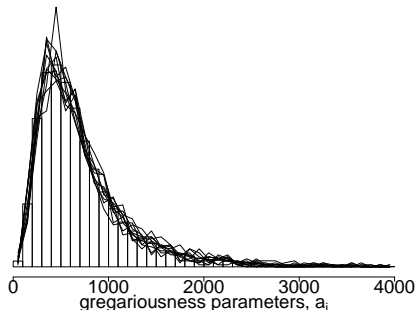
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Estimated distributions of network sizes for men and women

men



women

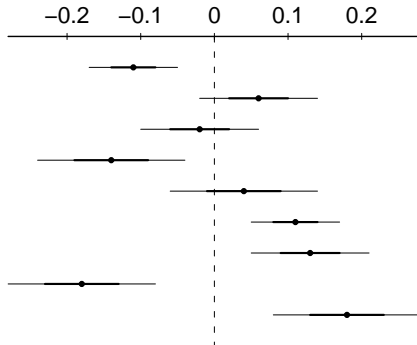


Regression of $\log(\text{gregariousness})$

Coefficient

Estimate

female
 nonwhite
 age < 30
 age > 65
 married
 college educated
 employed
 income < \$20,000
 income > \$80,000



Parameter estimates for the 32 subpopulations

► Subpopulations

- Names (Stephanie, Michael, etc.)
- Other groups (pilots, diabetics, etc.)

► Parameters

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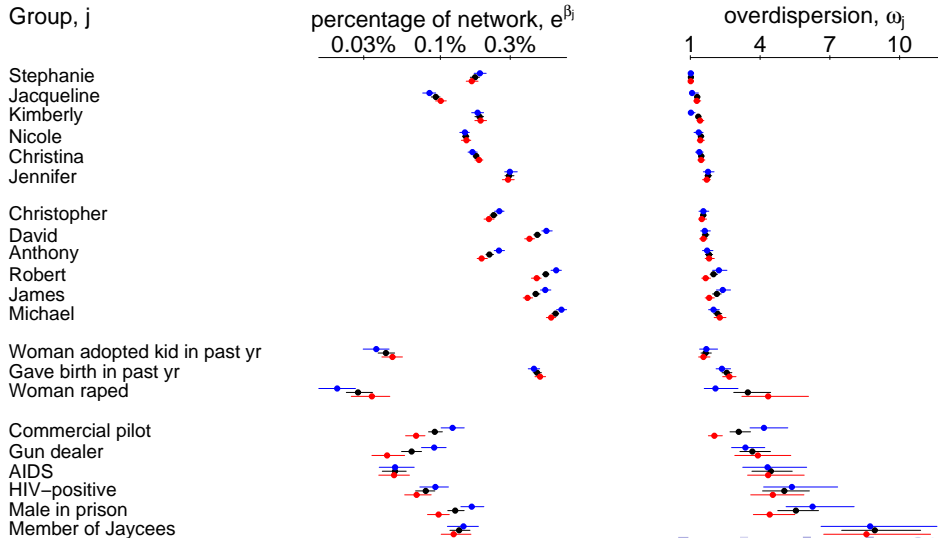
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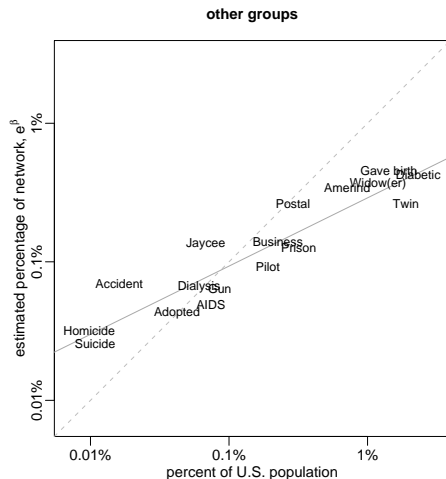
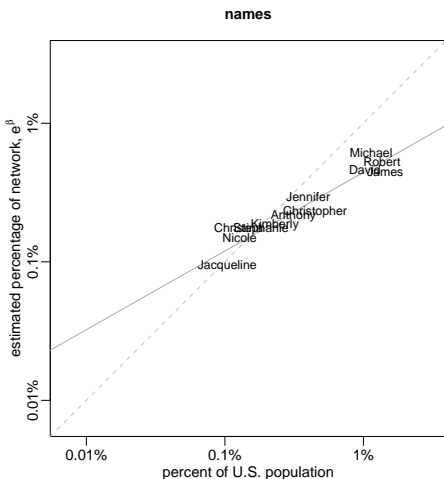
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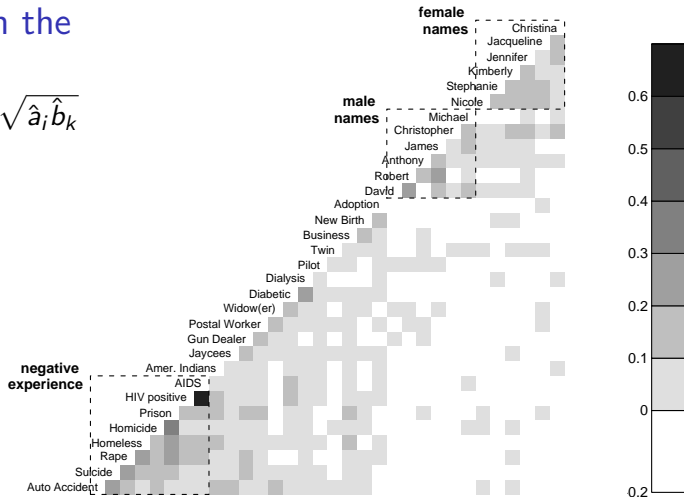
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Correlations in the residuals

$$r_{ik} = \sqrt{y_{ik}} - \sqrt{\hat{a}_i \hat{b}_k}$$



Regression of residuals for “How many prisoners do you know?”

Coefficient

Estimate

female

nonwhite

age < 30

age > 65

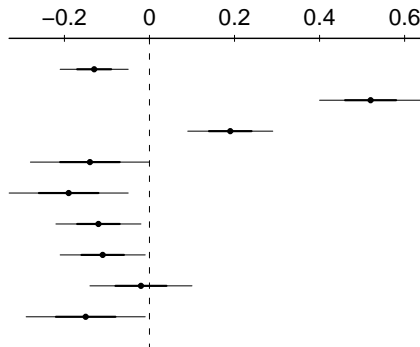
married

college educated

employed

income < \$20,000

income > \$80,000



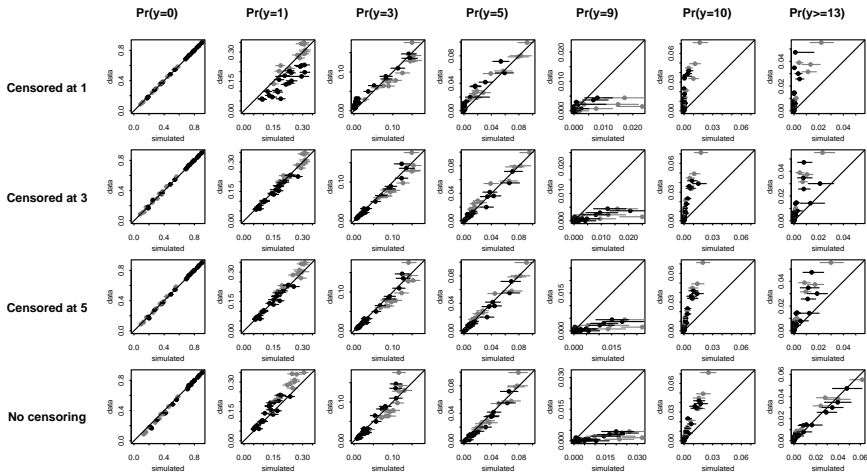
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Actual vs. simulated proportions of $y = 0, 1, \dots$



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- ▶ Model fit is good, not perfect
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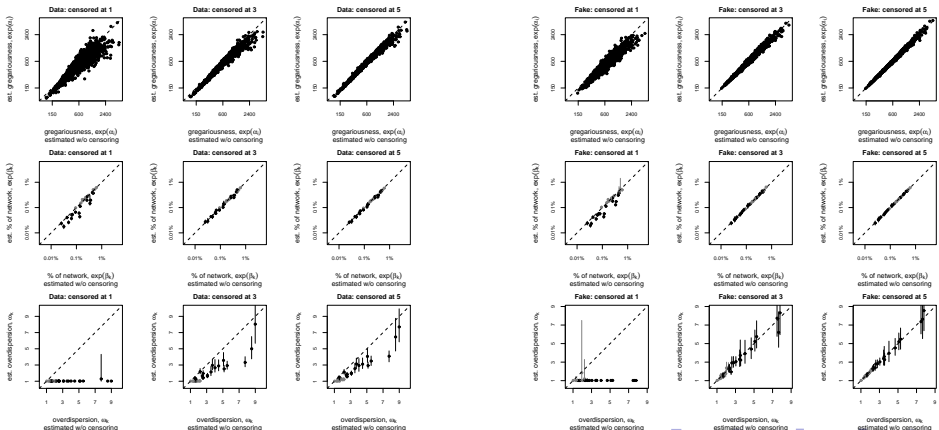
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Evaluation of inferences using fake data



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- ▶ Names are roughly uniformly distributed

- ▶ Some other groups show more structure

Bayesian models can model both global and local structure

Confidence

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- ▶ Parallel Metropolis updating for each vector α, β, ω
- ▶ Automated adaptive updating
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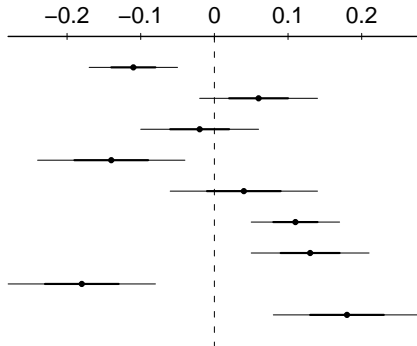
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Regression of $\log(\text{gregariousness})$: as a graph

Coefficient

Estimate

female
nonwhite
age < 30
age > 65
married
college educated
employed
income < \$20,000
income > \$80,000



Regression of $\log(\text{gregariousness})$: as a table

Coefficient	Estimate (s.e.)
female	-0.11 (0.03)
nonwhite	0.06 (0.04)
age < 30	-0.02 (0.04)
age > 65	-0.14 (0.05)
married	0.04 (0.05)
college educated	0.11 (0.03)
employed	0.13 (0.04)
income < \$20,000	-0.18 (0.05)
income > \$80,000	0.18 (0.05)

Running the demo

- ▶ How many Nicoles, Anthonys, lawyers, people robbed?
- ▶ Real-time data analysis
 - ▶ Entering in the data: 20 minutes
 - ▶ Running the program: 500 iterations (40 seconds), 1000 iterations (80 seconds)
 - ▶ Real-time data analysis: 35 minutes
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- ▶ Results for social network sizes, α
- ▶ Results for group sizes, β
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