# Expanded graphical models: Inference, Model comparison, Model checking, Fake-data debugging, and Model understanding

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# Generalized graphical models

- Expand graphical modeling to include:
  - Predictive model checking
  - Fake-data simulation
  - Scaffolding
- Common features:
  - Small changes to an existing fitted model
  - Comparisons of nodes between models

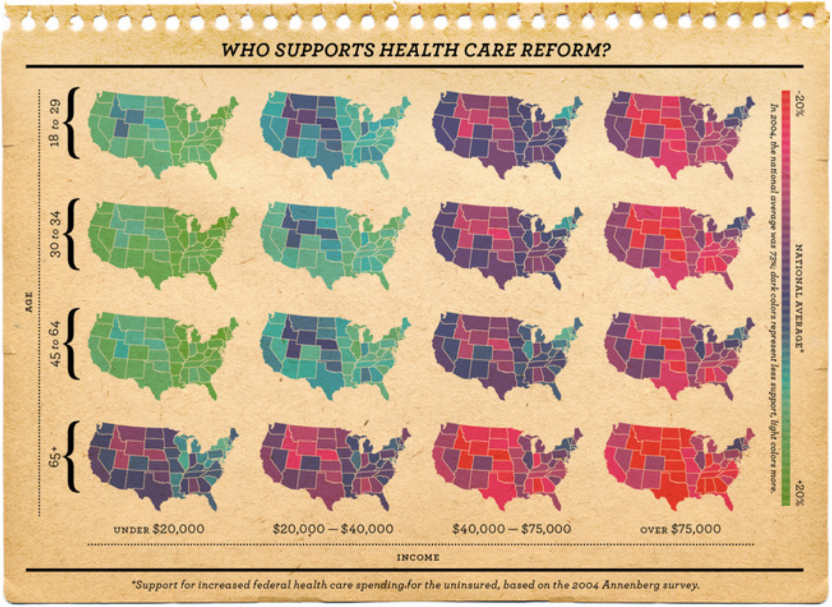
# Goals

- (applied) Building confidence in our computations and our models
- (methodological) Being able to do this routinely
- (theoretical): A unified framework for model building, model fitting, and model checking
- (computational): Implementing in a Bayesian computing environment such as stan

# 6 challenges in statistical modeling

- Setting up a realistic (i.e., complicated) model
- Regularization or partial pooling
- Fitting the model
- Checking the fit to data
- Confidence building
- Understanding the fitted model

# The models we're fitting

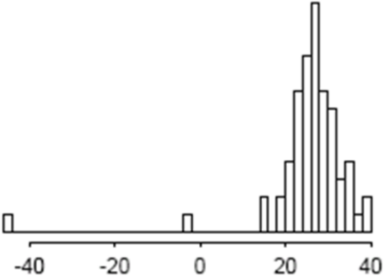


# Models for deep interactions

- Main effects, 2-way, 3-way, etc.
- Example: predicting public opinion given 4 age categories, 5 income categories, 50 states
- Also, group-level predictors (linear trends for age and income, previous voting patterns for states)
- Need a richer modeling language than this:
  - ```
    bglmer (y ~ z.age*z.inc*rvote.st + (z.age*z.inc | st) +
      (z.age*rvote.st | inc) + (z.inc*rvote.st | age) +
      (z.age | inc*st) + (z.inc | age*st) + (z.st |age*inc) +
      (1 | age*inc*st), family=binomial(link="logistic"))
    ```
  - No easy way to write this in Bugs or to program it oneself!
    ```

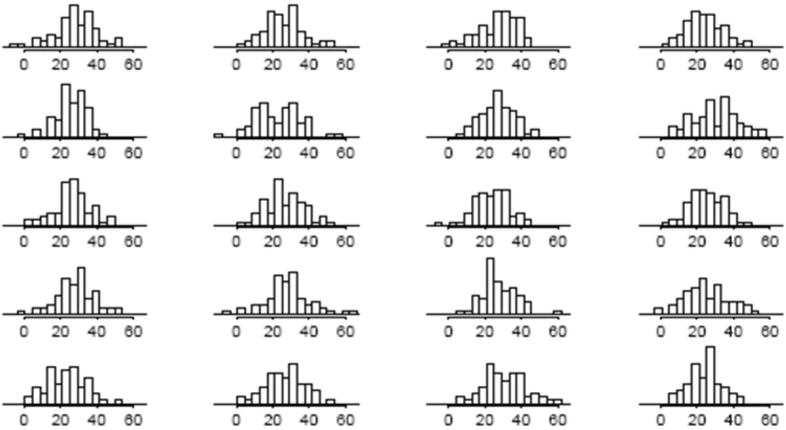
# Posterior predictive checking: 3 examples

Example 1: a normal distribution is fit to the following data:



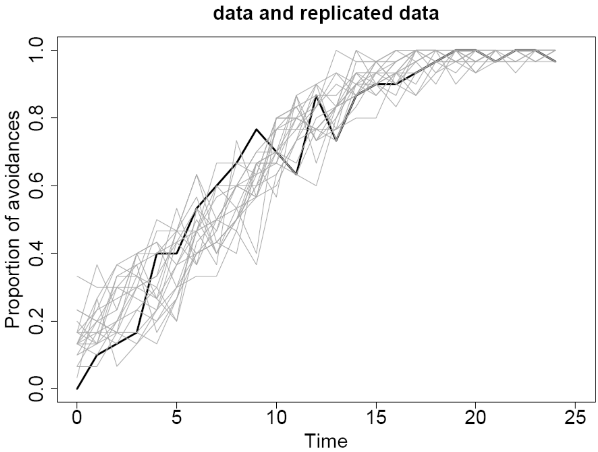
# Example 1 of 3: checking a fit to a univariate dataset

20 replicated datasets under the model:



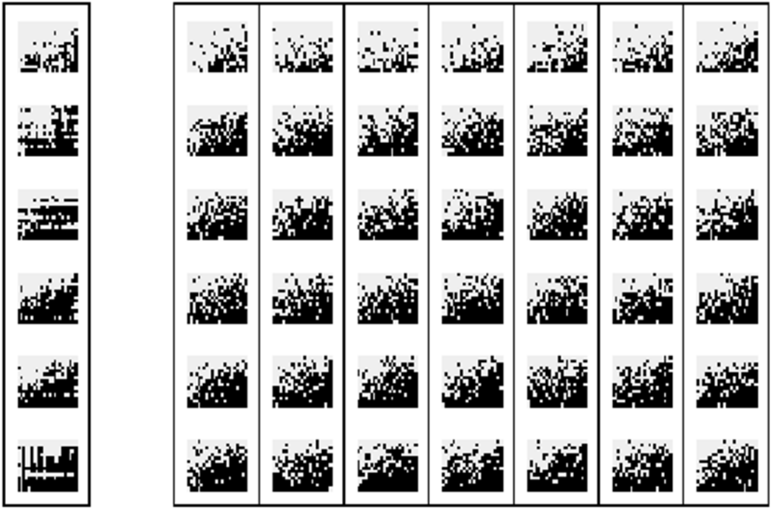
# Example 2: checking a model fit to data with time ordering

```
> plot (y, type="l")
> lines (y.rep)
```



# Example 3: checking a model with three-way structure

Data and 7 replications:



# Theoretical framework for predictive checking

- All our models are wrong
- What aspects of our models don't fit the data?
- Data and replicated data: $\theta \rightarrow y, y^{\mathrm{rep}}$
- Posterior predictive distribution, $p(y^{\mathrm{rep}}|y)$
- Computation:
  - Simulate $\theta$ from the posterior distribution, $p(\theta|y)$
  - Simulate $y^{\mathrm{rep}}$ from the predictive distribution, $p(y^{\mathrm{rep}}|\theta, y)$
  - Compare $y$ to the replicated datasets $y^{\mathrm{rep}}$
- The generalized graphical model:

```
M --> theta --> y
            \
             \
              y.rep
```

# Predictive checking and graphical models

- A posterior predictive check requires:
  - Set of conditioning variables $\theta$
  - Set of fixed design variables $X$ (e.g., sample size)
  - Test variable $T(y)$ (more generally, $T(X, y, \theta)$)
- Simulating posterior predictive replications is a fundamental operation in graphical models
- Requires a new node, $y^{\mathrm{rep}}$, whose distribution is implied by the existing model

# Fake-data debugging

- Sample $\theta^{\text{pretend}}$ from the prior distribution $p(\theta)$
- Sample $y$ from the model $p(y|\theta^{|\text{pretend}})$
- Perform Bayesian inference, simulations from $p(\theta|y)$
- Check calibration of posterior means, predictive intervals, etc. compared to $\theta^{\text{pretend}}$ (Cook, Gelman, and Rubin, 2007)
- Fake-data simulation is a fundamental operation in graphical models
- $\theta^{\text{pretend}}$ is a new node

```
M --> th.pretend --> y
  \                /
    \            /
      \        /
         theta
```

# Generalized graphical models

- Step 0 (already done): Expressing a statistical model as a graph; Bayesian computation on the graph
- Step 1: Graph of models
  - Each model is a node of this super-graph
  - Two models are connected if they differ by only one feature (adding/removing a variable, allowing a parameter to vary by group, adding/removing a grouping factor, changing a probability distribution or link function, ...)
- Step 2: Integrated graph
  - Nodes within models are linked within a larger graph
  - All models coexist
  - Analogy to computational method of parallel tempering

# Automatic posterior predictive checking

- Example in Bugs:

```
for (i in 1:n){
  y[i] ~ dnorm (y.hat[i], tau.y)
  y.rep[i] <- dnorm (y.hat[i], tau.y)
  . . .
```

- But $y^{\mathrm{rep}}$ should be included automatically
- Implicit graphical structure for model checking: $y \leftarrow \theta \rightarrow y^{\mathrm{rep}}$

# Predictive checking and fake-data debugging

- Ideal of model checking or debugging in stan, Bugs, etc.:
  - On/off switch for each node: is it conditioned on or averaged over?
  - Specify a test summary (numerical or graphical) of data and parameters
  - Various off-the-shelf test summaries will be available
- Design of data collection is integrated with graphical modeling

# The network of models

- Each node is itself a graphical model
- Common parameters in neighboring models are linked
- Computations in the network:
  - Inference within a model
  - Inference among models (model comparison, averaging, and expansion)
  - Model checking
  - Fake-data debugging
  - Model understanding (exploratory model analysis)

# Summary and future directions

- Generalized graphical models:

```
M --> theta --> y              M --> theta --> y
            \                    \
             \          or        \
              y.rep                theta.rep --> y.rep
```

- All these quantities—$\theta, y, y^{\text{rep}}$—exist together
- Model checking can be done systematically
- All is completely Bayesian—there is no "double use of data"!
- A theoretical and computational unification of different aspects of statistical practice