Posterior Predictive Checking and Generalized Graphical Models

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Generalized graphical models

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  - Fake-data simulation
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- Common features:
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  - Comparisons of nodes between models

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- (methodological) Being able to do this routinely
- (theoretical): A unified framework for model building, model fitting, and model checking
- (computational): Implementing in a Bugs-like language

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- $\begin{pmatrix} \alpha_j \\ \beta_j \end{pmatrix} \sim \mathrm{N}\left(\begin{pmatrix} \mu_\alpha \\ \mu_\beta \end{pmatrix}, \begin{pmatrix} \sigma_\alpha^2 & \rho\sigma_\alpha\sigma_\beta \\ \rho\sigma_\alpha\sigma_\beta & \sigma_\beta^2 \end{pmatrix} \right)$, for $j = 1, \ldots, J$

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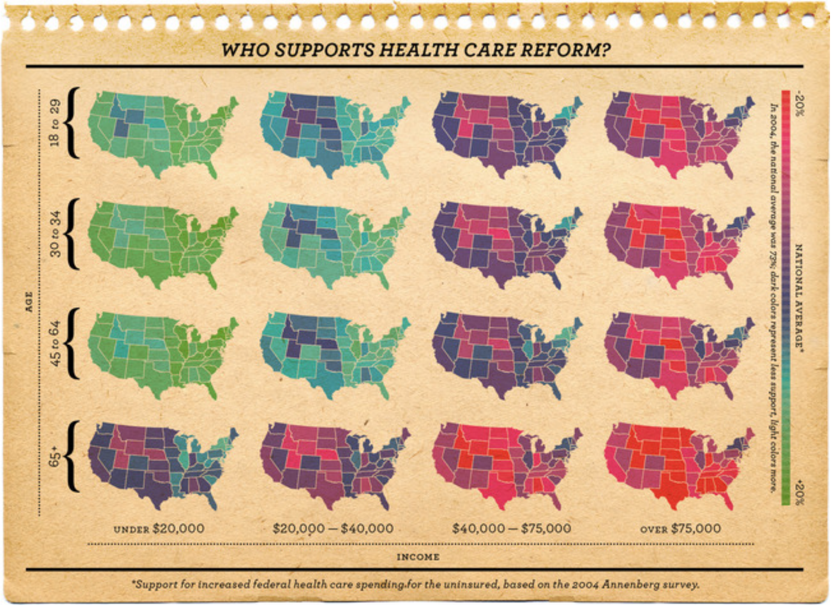
- Also can have group-level predictors and nonnested grouping factors

Application: public opinion in population subgroups



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  - ```
    glmer (y ~ z.age*z.inc*rvote.st + (z.age*z.inc | st) +
      (z.age*rvote.st | inc) + (z.inc*rvote.st | age) +
      (z.age | inc*st) + (z.inc | age*st) + (z.st |age*inc) +
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    ```
    ```

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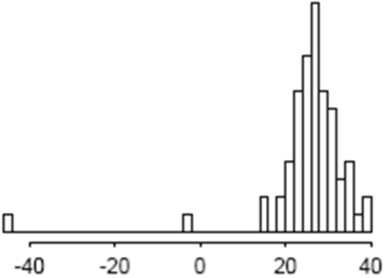
  - No easy way to write this in Bugs or to program it oneself!

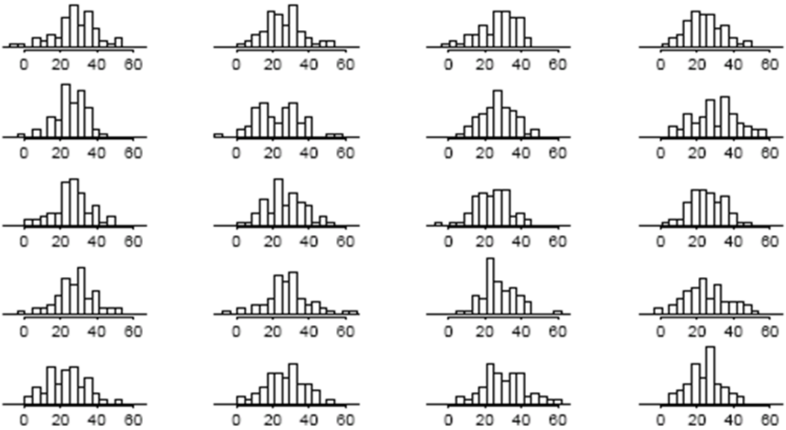
Posterior predictive checking: 3 examples

Example 1: a normal distribution is fit to the following data:



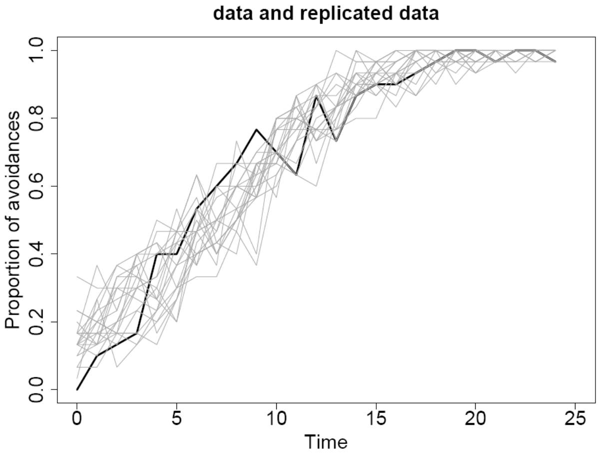
Example 1 of 3: checking a fit to a univariate dataset

20 replicated datasets under the model:



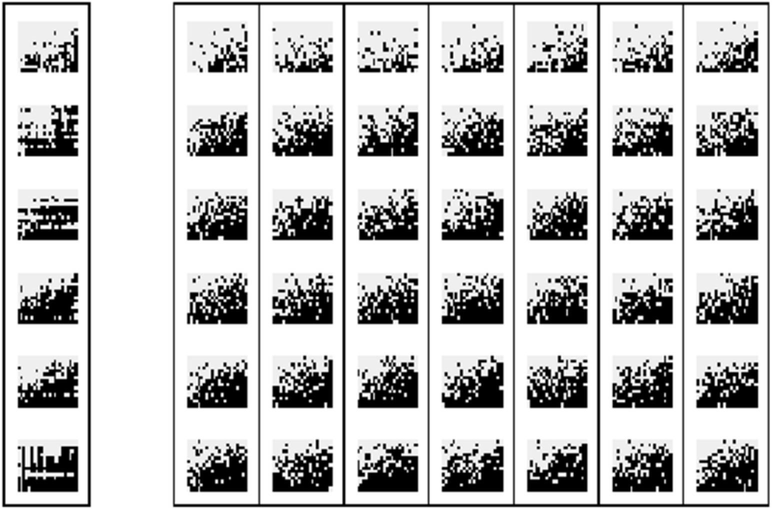
Example 2: checking a model fit to data with time ordering

```
> plot (y, type="l")
> lines (y.rep)
```



Example 3: checking a model with three-way structure

Data and 7 replications:



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- The generalized graphical model:

```
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           \
            \
             y.rep
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- Voilà la connection d'analyse exploratoire des données (EDA) et la presentation visuelle
- Les "*p*-values" sont les moins importants choses dans la vérification posterior predictive!

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  - Connection to graphical models!

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- Requires a new node, y^{rep}, whose distribution is implied by the existing model

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  - General procedure in Cook, Gelman, and Rubin (2007)
- Fake-data simulation is a fundamental operation in graphical models
- θ^{true} is a new node

```
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  \                 /
    \             /
      \         /
         theta
```

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  - Analogy to computational method of parallel tempering

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- Example:

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for (i in 1:n){
  y[i] ~ dnorm (y.hat[i], tau.y)
  y.hat[i] <- a[state[i]] + b[state[i]]*x[i]
  e.y[i] <- y[i] - y.hat[i]
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tau.y <- pow(sigma.y, -2)
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- Also, instead of y.hat, sigma.y, e.y, we want a more general "operator" notation, for example E(y), sd(y), error(y)

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- Implicit graphical structure for model checking: $y \leftarrow \theta \rightarrow y^{\mathrm{rep}}$

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  - Various off-the-shelf test summaries will be available
- Design of data collection is integrated with graphical modeling

Résumé et les directions vers l'avenir

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```
M --> theta --> y                 M --> theta --> y
            \                           \
             \          ou               \
              y.rep                       theta.rep --> y.rep
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M --> theta --> y                 M --> theta --> y
            \                         \
             \          ou             \
              y.rep                     theta.rep --> y.rep
```

- Tous les quantitées—$\theta, y, y^{\mathrm{rep}}$—existent ensemble

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Résumé et les directions vers l'avenir

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