## Contents

Preface		xiii	
Part I: Fundamentals of Bayesian Inference		1	
1	Prol	pability and inference	3
	1.1	The three steps of Bayesian data analysis	3
	1.2	General notation for statistical inference	4
	1.3	Bayesian inference	6
	1.4	Discrete probability examples: genetics and spell checking	8
	1.5	Probability as a measure of uncertainty	11
	1.6	Example of probability assignment: football point spreads	13
	1.7	Example: estimating the accuracy of record linkage	16
	1.8	Some useful results from probability theory	19
	1.9	Computation and software	22
	1.10	Bayesian inference in applied statistics	24
	1.11	Bibliographic note	25
	1.12	Exercises	27
2	Single-parameter models		29
	2.1	Estimating a probability from binomial data	29
	2.2	Posterior as compromise between data and prior information	32
	2.3	Summarizing posterior inference	32
	2.4	Informative prior distributions	34
	2.5	Estimating a normal mean with known variance	39
	2.6	Other standard single-parameter models	42
	2.7	Example: informative prior distribution for cancer rates	47
	2.8	Noninformative prior distributions	51
	2.9	Weakly informative prior distributions	55
	2.10	Bibliographic note	56
	2.11	Exercises	57
3	Intr	oduction to multiparameter models	63
	3.1	Averaging over 'nuisance parameters'	63
	3.2	Normal data with a noninformative prior distribution	64
	3.3	Normal data with a conjugate prior distribution	67
	3.4	Multinomial model for categorical data	69
	3.5	Multivariate normal model with known variance	70
	3.6	Multivariate normal with unknown mean and variance	72
	3.7	Example: analysis of a bioassay experiment	74
	3.8	Summary of elementary modeling and computation	78
	3.9	Bibliographic note	78
	3.10	Evercises	70

viii	CONTENTS
V111	CONTENTS

4	<b>Asy</b> : 4.1	mptotics and connections to non-Bayesian approaches  Normal approximations to the posterior distribution	<b>83</b> 83
	4.2	Large-sample theory	87
	4.3	Counterexamples to the theorems	89
	4.4	Frequency evaluations of Bayesian inferences	91
	4.5	Bayesian interpretations of other statistical methods	92
	4.6	Bibliographic note	97
	4.7	Exercises	98
5	Hier	earchical models	101
	5.1	Constructing a parameterized prior distribution	102
	5.2	Exchangeability and setting up hierarchical models	104
	5.3	Fully Bayesian analysis of conjugate hierarchical models	108
	5.4	Estimating exchangeable parameters from a normal model	113
	5.5	Example: parallel experiments in eight schools	119
	5.6	Hierarchical modeling applied to a meta-analysis	124
	5.7	Weakly informative priors for hierarchical variance parameters	128
	$5.8 \\ 5.9$	Bibliographic note Exercises	132 134
Pa	art II	: Fundamentals of Bayesian Data Analysis	139
6	Mod	lel checking	141
U	6.1	The place of model checking in applied Bayesian statistics	141
	6.2	Do the inferences from the model make sense?	142
	6.3	Posterior predictive checking	143
	6.4	Graphical posterior predictive checks	153
	6.5	Model checking for the educational testing example	159
	6.6	Bibliographic note	161
	6.7	Exercises	163
7	Eval	uating, comparing, and expanding models	165
	7.1	Measures of predictive accuracy	166
	7.2	Information criteria and cross-validation	169
	7.3	Model comparison based on predictive performance	178
	7.4	Model comparison using Bayes factors	182
	7.5	Continuous model expansion	184
	7.6		187
	7.7 7.8	Bibliographic note Exercises	192 193
_			
8		leling accounting for data collection	197
	8.1	Bayesian inference requires a model for data collection	197
	8.2	Data-collection models and ignorability	199
	8.3 8.4	Sample surveys Designed experiments	$     \begin{array}{r}       205 \\       214   \end{array} $
	8.5	Sensitivity and the role of randomization	214
	8.6	Observational studies	220
	8.7	Censoring and truncation	224
	8.8	Discussion	$\frac{224}{229}$
	8.9	Bibliographic note	229
		Exercises	230

C	ONTE	ENTS	ix
9	Deci	ision analysis	237
	9.1	Bayesian decision theory in different contexts	237
	9.2	Using regression predictions: incentives for telephone surveys	239
	9.3	Multistage decision making: medical screening	245
	9.4	Hierarchical decision analysis for radon measurement	246
	9.5	Personal vs. institutional decision analysis	256
	9.6	Bibliographic note	257
	9.7	Exercises	257
Pa	art II	I: Advanced Computation	<b>25</b> 9
10	Intr	oduction to Bayesian computation	261
	10.1	Numerical integration	261
	10.2	Distributional approximations	262
	10.3	Direct simulation and rejection sampling	263
	10.4	Importance sampling	265
		How many simulation draws are needed?	267
		Computing environments	268
		Debugging Bayesian computing	270
		Bibliographic note	271
	10.9	Exercises	272
11	Basi	cs of Markov chain simulation	<b>27</b> 5
	11.1	Gibbs sampler	276
	11.2	Metropolis and Metropolis-Hastings algorithms	278
	11.3	Using Gibbs and Metropolis as building blocks	280
	11.4	Inference and assessing convergence	281
	11.5	Effective number of simulation draws	286
	11.6	Example: hierarchical normal model	288
	11.7	Bibliographic note	291
	11.8	Exercises	291
12		nputationally efficient Markov chain simulation	293
		Efficient Gibbs samplers	293
		Efficient Metropolis jumping rules	295
		Further extensions to Gibbs and Metropolis	297
		Hamiltonian Monte Carlo	300
		Hamiltonian dynamics for a simple hierarchical model	305
		Stan: developing a computing environment	307
		Bibliographic note	308
	12.8	Exercises	309
13		lal and distributional approximations	311
		Finding posterior modes	311
		Boundary-avoiding priors for modal summaries	313
		Normal and related mixture approximations	318
		Finding marginal posterior modes using EM	320
		Approximating conditional and marginal posterior densities	325
		Example: hierarchical normal model (continued)	326
		Variational inference	331
	13.8	Expectation propagation	338

343

13.9 Other approximations

X	CONTENTS

13.1	0 Unknown normalizing factors	345
13.1	1 Bibliographic note	348
13.1	2 Exercises	349
Part I	V: Regression Models	351
	oduction to regression models	353
	Conditional modeling	353
	Bayesian analysis of the classical regression model	354
	Regression for causal inference: incumbency in congressional elections	358
	Goals of regression analysis	364
	Assembling the matrix of explanatory variables	365
	Regularization and dimension reduction for multiple predictors	367
	Unequal variances and correlations	369
	Including numerical prior information	376
	Bibliographic note	378
14.1	0 Exercises	378
	rarchical linear models	381
	Regression coefficients exchangeable in batches	382
	Example: forecasting U.S. presidential elections	383
	Interpreting a normal prior distribution as additional data	388
	Varying intercepts and slopes	390
	Computation: batching and transformation	392
	Analysis of variance and the batching of coefficients	395
	Hierarchical models for batches of variance components	398
	Bibliographic note	400
15.9	Exercises	402
	eralized linear models	405
	Standard generalized linear model likelihoods	406
	Working with generalized linear models	407
	Weakly informative priors for logistic regression	412
	Example: hierarchical Poisson regression for police stops	420
	Example: hierarchical logistic regression for political opinions	422
	Models for multivariate and multinomial responses	423
	Loglinear models for multivariate discrete data	428
	Bibliographic note	431
16.9	Exercises	432
	dels for robust inference	435
	Aspects of robustness	435
	Overdispersed versions of standard probability models	437
	Posterior inference and computation	439
	Robust inference and sensitivity analysis for the eight schools	441
	Robust regression using t-distributed errors	444
	Bibliographic note	445
17.7	Exercises	446

CONT	TENTS	xi
18 Mo	odels for missing data	449
18.	1 Notation	449
18.	2 Multiple imputation	451
18.	3 Missing data in the multivariate normal and $t$ models	454
18.	4 Example: multiple imputation for a series of polls	456
	5 Missing values with counted data	462
18.	6 Example: an opinion poll in Slovenia	463
	7 Bibliographic note	466
18.	8 Exercises	467
Part '	V: Nonlinear and Nonparametric Models	469
19 Pa	rametric nonlinear models	471
19.	1 Example: serial dilution assay	471
19.	2 Example: population toxicokinetics	477
19.	3 Bibliographic note	485
19.	4 Exercises	486
20 Ba	sis function models	487
20.	1 Splines and weighted sums of basis functions	487
20.	2 Basis selection and shrinkage of coefficients	490
20.	3 Non-normal models and multivariate regression surfaces	494
20.	4 Bibliographic note	498
20.	5 Exercises	498
21 Ga	aussian process models	501
21.	1 Gaussian process regression	501
21.	2 Example: birthdays and birthdates	505
21.	3 Latent Gaussian process models	510
21.	4 Functional data analysis	512
21.	5 Density estimation and regression	513
21.	6 Bibliographic note	516
21.	7 Exercises	516
22 Fi	nite mixture models	519
22.	1 Setting up and interpreting mixture models	519
22.	2 Example: reaction times and schizophrenia	524
22.	3 Label switching and posterior computation	533
22.	4 Unspecified number of mixture components	536
22.	5 Mixture models for classification and regression	539
22.	6 Bibliographic note	542
22.	7 Exercises	543
23 Di	richlet process models	545
	1 Bayesian histograms	545
	2 Dirichlet process prior distributions	546
	3 Dirichlet process mixtures	549
	4 Beyond density estimation	557
	5 Hierarchical dependence	560
	6 Density regression	568
	7 Bibliographic note	571
	8 Exercises	573

xii			CONTENTS
A		dard probability distributions Continuous distributions	<b>575</b> 575
		Discrete distributions	583
	A.3	Bibliographic note	584
В	Outl	ine of proofs of limit theorems	585
	B.1	Bibliographic note	588
$\mathbf{C}$	Computation in R and Stan		589
	C.1	Getting started with R and Stan	589
	C.2	Fitting a hierarchical model in Stan	589
	C.3	Direct simulation, Gibbs, and Metropolis in R	594
	C.4	Programming Hamiltonian Monte Carlo in R	601
	C.5	Further comments on computation	605
	C.6	Bibliographic note	606
References			607
Αı	$_{ m ithor}$	Index	641
Su	ıbject	Index	649