Separating Signal from Background

Bodhisattva Sen[1]
Department of Statistics
Columbia University

February 6, 2009

[1] Collaborative work with Matthew Walker, Mario Mateo & Michael Woodroofe

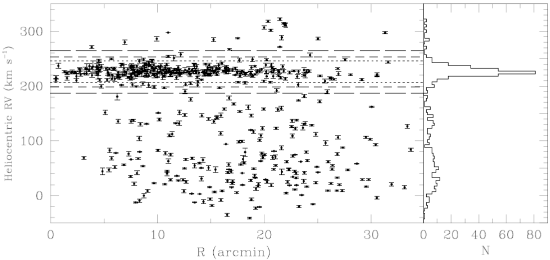
Outline

Problem

- Most astronomical data sets are polluted to some extent by *foreground/background* objects (“contaminants/noise”) that can be difficult to distinguish from objects of interest (“member/signal”)
- Contaminants may have the same apparent magnitudes, colors, and even velocities as members
- How do you *separate* out the *“signal”* stars?
- We develop an algorithm for evaluating membership (estimating *parameters* & *probability* of an object belonging to the member population)

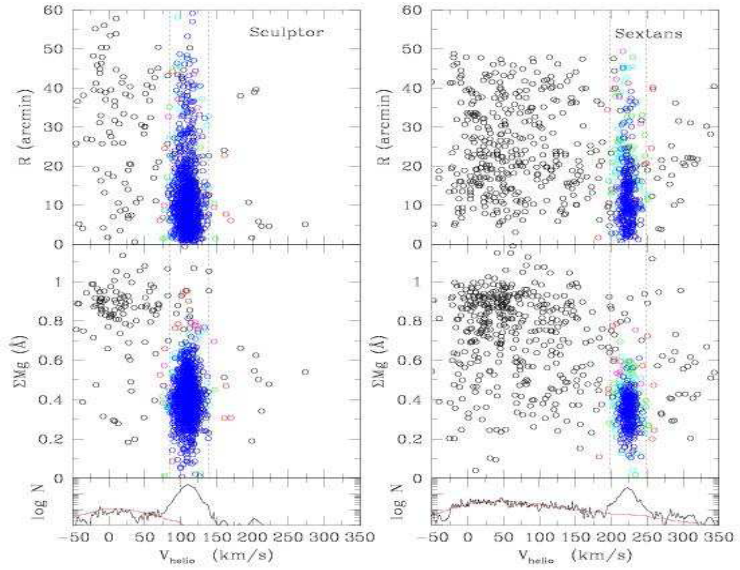
Example

- Data on stars in nearby dwarf spheroidal (dSph) galaxies
- Data: $(X_{1i}, X_{2i}, V_{3i}, \sigma_i, \Sigma Mg_i, ...)$
- Velocity samples suffer from *contamination* by foreground Milky Way stars



Approach

- Our method is based on the *Expectation-Maximization* (EM) algorithm
- We assign *parametric distributions* to the observables; derived from the underlying physics in most cases
- The EM algorithm provides *estimates* of the unknown parameters (mean velocity, velocity dispersion, etc.)
- Also, *probability* of each star belonging to the signal population


Sculptor
Sextans
R (arcmin)
ΣMg (Å)
log N
V helio (km/s)
-50 0 50 100 150 200 250 300 350

A toy example

- Suppose $N \sim$ Poisson$(b + s)$ is the number of stars observed
- $s =$ rate for observing a *member* star
- $b =$ the *foreground* rate
- Given $N = n$, we have $W_1, \ldots, W_n \sim f_{b,s}$ where data $\{W_i = (X_{1i}, X_{2i}, V_{3i}, \sigma_i)\}_{i=1}^n$, and $f_{b,s}(w) = \frac{bf_b(w)+sf_s(w)}{b+s}$
- We assume that f_b and f_s are *parameterized* (modeled by the underlying physics) probability densities

For Galaxy stars

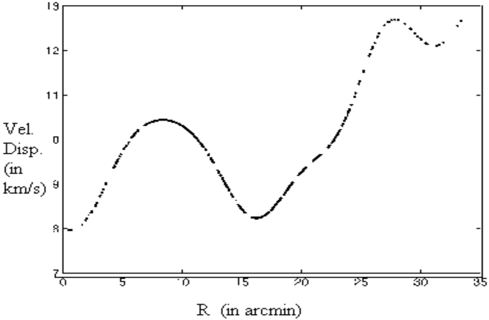
- The stellar density (number of stars per unit area) falls *exponentially with radius*, R
- The distribution of velocity given position is assumed to be *normal with mean* μ *and variance* $\sigma^2 + \sigma_i^2$

For Foreground stars

- The density is *uniform over the field of view*
- The distribution of velocities V_{3i} is *independent* of position (X_{1i}, X_{2i})
- We adopt V_{3i} from the *Besanćon* Milky Way model (Robin et al. 2003), which specifies velocity distributions of Milky Way stars along a given line of sight

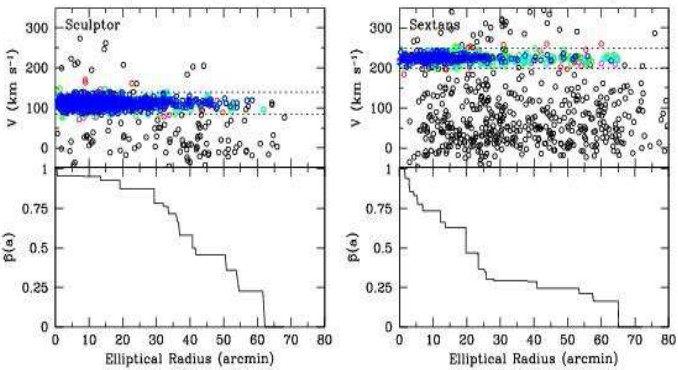
Extension: Scenario I

- Introduce a *non-parametric* component
- *Velocity dispersion* was assumed constant; now can model it as a function of *projected radius* R
- Needs a tuning parameter to find $\sigma(r)$



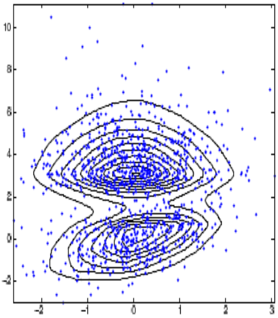
Scenario II

- Do not assume *exponential* density profile
- Assume that as you move from the center of the galaxy, the chance of observing a “member” star *decreases*



A further extension

- Mixture model $f(x) = \sum_{j=1}^{k} \pi_j f_j(x)$; $\sum_{j=1}^{k} \pi_j = 1$
- Model $f_1, f_2, \ldots, f_k$ as *log-concave* densities on $\mathbb{R}^p$
- No *Tuning* parameter required – completely *non-parametric*



Model

- Suppose $N \sim$ Poisson$(b+s)$ is the number of stars observed
- $s =$ rate for observing a *member* star
- $b =$ the *foreground* rate
- Given $N = n$, we have $W_1, \ldots, W_n \sim f_{b,s}$ where data $\{W_i = (X_{1i}, X_{2i}, V_{3i}, \sigma_i, \Sigma Mg_i, \ldots)\}_{i=1}^n$, and $f_{b,s}(w) = \frac{bf_b(w)+sf_s(w)}{b+s}$
- We assume that f_b and f_s are *parameterized* (modeled by the underlying physics) probability densities

The Likelihood

- Let *Y_i be the indicator of a foreground star*, i.e., $Y_i = 1$ if the i'th star is a foreground star, and $Y_i = 0$ otherwise
- Note that *Y_i's are i.i.d. Bernoulli($\frac{b}{b+s}$)*. Let $\mathbf{Z} = (\mathbf{W}, \mathbf{Y}, N)$ be the *complete* data [where $\mathbf{W} = (W_1, W_2, \ldots, W_n)$ and $\mathbf{Y} = (Y_1, Y_2, \ldots, Y_n)$]
- The *likelihood* for the *complete data* can be written as

$$L^C(\beta) = e^{-(b+s)} \frac{(b+s)^N}{N!} \prod_{i=1}^{N} \left\{ \frac{b\, f_b(W_i)}{b+s} \right\}^{Y_i} \left\{ \frac{s\, f_s(W_i)}{b+s} \right\}^{1-Y_i}$$

Algorithm

- Start with some initial estimates of the parameter β [in our simple example $\beta = (s, b, \mu, \sigma^2, \ldots)$]

E-step:

- Evaluates the *expectation of the log-likelihood* given the *observed* data under the current estimates of the unknown parameters
- Evaluate $Q(\beta, \widehat{\beta}_n) = E_{\widehat{\beta}_n}[l(\beta)|\mathbf{W}, N]$

M-step:

- *Maximizes the expectation* $Q(\beta, \widehat{\beta}_n)$ with respect to β

- Iterate until the estimates *stabilize* (which is guaranteed!)

Summary: EM algorithm with mixture models

- *Estimates* of unknown parameters
- Estimated *probability* that the *i*'th star is a member
- Allows *flexible* modeling (non-parametric) of data
- All we need is to form the *likelihood*!

References

- Walker et al. (2008): to appear in *Astronomical Journal*
- Sen, B., et al. (2008): to appear in *Statis. Sinica*
- Cule, M. L. (2008): submitted