Imaging, Communications and Finance: Stochastic Modeling of Real-world Problems

Conference in honor of Lawrence A. Shepp

June 24-25, 2011
Columbia University
New York, NY
To celebrate the 75th birthday of Dr. Lawrence Shepp we highlight the many contributions to stochastic modeling of real-world problems that he has made throughout his career.

Statistics and probability have come to play an increasingly important role in a variety of interdisciplinary research areas including neuroscience, genetics, physics, finance and engineering. The goal of this conference is to focus on a number of important and emerging interdisciplinary areas of research, such as imaging, telecommunications and finance. Though the individual topic areas are diverse, a unifying theme for the conference is the use of probabilistic, combinatorial, and statistical analysis of models for problems arising in the real world. The conference will highlight important contributions already made through the use of statistics and probability, including the development of new reconstruction algorithms for medical imaging, stochastic models in risk analysis and finance, and methods for analyzing complicated network data. In addition, we will also work on identifying emerging issues where statistics and probability promise to play an important role in the future.

The conference is organized by the Department of Statistics and the Center for Applied Probability at Columbia University and is supported by an NSF DMS grant.
SCHEDULE

All talks will be held in Room 501 Northwest Corner Building (NCB) on the Morningside Campus of Columbia University. The poster session will be held in Room 602 NCB. Breakfast and Coffee breaks will be served on the 6th floor of NCD, where the registration is also located.

Friday June 24, 2011

8:30 - 8:50  Breakfast and Registration
8:50 - 9:00  Opening Statements

Session I  Chair: Zhiliang Ying, Columbia University
9:00 - 9:40  Ron Graham, University of California San Diego
9:40 - 10:20 Srinivaa Varadhan, New York University
10:20 - 10:40 Coffee Break
10:40 - 11:20 Peter Winkler, Dartmouth College
11:20 - 12:00 Mike Steele, University of Pennsylvania

12:00 - 2:00  Lunch on own

Session II  Chair: Martin Lindquist, Columbia University
2:00 - 2:40  Ioannis Karatzas Columbia University
2:40 - 3:20  Mike Harrison, Stanford University
3:20 - 3:40  Coffee Break
3:40 - 4:20  Tze Leung Lai, Stanford University
4:20 - 5:00  Xin Guo, University of California Berkeley

5:00 - 6:00  Poster Session in 602 NCB
6:00 –  Dinner Banquet at Faculty Club
Saturday June 25, 2011

8:30 - 9:00  Breakfast

Session III  Chair: Martin Lindquist, Columbia University

9:00 - 9:40  Z.H. Cho, Gachon University of Medicine and Science
9:40 - 10:20 Charles Epstein, University of Pennsylvania

10:20 - 10:40  Coffee Break

10:40 - 11:20  Ed Coffman, Columbia University
11:20 - 12:00 Cun-Hui Zhang, Rutgers University

12:00 - 1:40  Lunch on own

Session IV  Chair: Cun-Hui Zhang, Rutgers University

1:40 - 2:20  Larry Brown, University of Pennsylvania
2:20 - 3:00  David Siegmund, Stanford University

3:00 - 3:20  Coffee Break

3:20 - 4:00  Ed George, University of Pennsylvania
4:00 - 4:40  Albert Shiryaev, Steklov Mathematical Institute

Session V  Chair: Zhiliang Ying, Columbia University

4:40 - 5:30  Larry Shepp, University of Pennsylvania
INVITED TALKS

Larry Brown
Department of Statistics
University of Pennsylvania

Stationary Infinitely-Divisible Markov Processes with Non-negative Integer Values

We characterize all stationary time-reversible Markov processes whose finite-dimensional marginal distributions (of all orders) are infinitely divisible. Aside from two trivial cases (iid and constant), every such process in both discrete and continuous time is a branching process with Poisson or Negative Binomial marginal distributions and a specific bivariate distribution at pairs of times. (joint work with Robert Wolpert)

Z.H. Cho
Neuroscience Research Institute
Gachon University of Medicine and Science

The Birth of 3-D Image Reconstruction - From CT to PET and MRI

The birth of 3-D reconstruction in the early 70’s has given rise to many changes in our life, including medical imaging techniques such as CT, PET and MRI. Now more than a hundred thousand CT, PET and MRI scanners are used every day for the diagnosis and treatment of various human diseases from the detection of cancerous tumors to cognitive neuroscience research. Undoubtedly, one of the most prominent contributions to the area of medical imaging is the development of the mathematics and algorithm for Reconstruction Tomography or 3-D Image Reconstruction Mathematics. Larry Shepp stands as one of the key contributors to the field since the inception of the idea and often we hear about Shepp’s Algorithm. This talk will review some of the historical aspects of the development of the 3-D reconstruction algorithm and its proliferation to what we see today, including PET-MRI and its applications to the neuroscience research.
Ed Coffman
Departments of Electrical Engineering and Computer Science
Columbia University

Stochastic Fragmentation in the Real World: Seminal Research of L.A. Shepp

Stochastic fragmentation is created by random arrivals and departures of items occupying some arena of available space. A principal application driving L. A. Shepp’s original stochastic analysis of fragmentation processes was dynamic storage allocation, a problem in one dimension where the items are files to be stored in computer memory. Shepp’s work on dynamic fragmentation in that context was arguably the field’s first rigorous treatment leading to explicit formulas, results that provided important insights into the nature of storage fragmentation. We review this work and the substantial impact it has had on subsequent developments within both the mathematics and computer science communities.

Charles Epstein
Department of Mathematics
University of Pennsylvania

Analytic Foundations for Diffusion Models in Population Genetics

The infinite population limits of standard Markov Chain models in population genetics are diffusions that take place on polyhedral domains. The generators are second order operators with second order terms degenerating along the boundary of the domain. I will discuss recent joint work with Rafe Mazzeo establishing existence and uniqueness results for operators of this type.
Ed George  
Department of Statistics  
University of Pennsylvania  

High Dimensional Predictive Inference

Let $X \mid \mu \sim N_p(\mu, v_x I)$ and $Y \mid \mu \sim N_p(\mu, v_y I)$ be independent $p$-dimensional multivariate normal vectors with common unknown mean $\mu$. Based on only observing $X = x$, we consider the problem of obtaining a predictive density $\hat{p}(y \mid x)$ for $Y$ that is close to $p(y \mid \mu)$ as measured by expected Kullback-Leibler loss. This is the predictive version of the general problem of estimating $\mu$ under quadratic loss, and we see that a strikingly parallel theory exists for addressing it.

To begin with, a natural “straw man” procedure for this problem is the (formal) Bayes predictive density $\hat{p}_U(y \mid x)$ under the uniform prior $\pi_U(\mu) \equiv 1$, which is best invariant and minimax. It turns out that there are wide classes procedures that dominate $\hat{p}_U(y \mid x)$ including Bayes predictive densities under superharmonic priors. Indeed, any Bayes predictive density will be minimax if it is obtained by a prior yielding a marginal that is superharmonic or whose square root is superharmonic. For the characterization of admissible procedures for this problem, the class of all generalized Bayes rules here is seen to form a complete class, and easily interpretable conditions are seen to be sufficient for the admissibility of a formal Bayes rule.

Moving on to the multiple regression setting where we observe $X \sim N_m(A\beta, \sigma^2 I)$ and would like to estimate the predictive density $p(y \mid \beta)$ of a future $Y \sim N_n(B\beta, \sigma^2 I)$, our results are seen to extend naturally. Going further, we address the situation where there is model uncertainty and only an unknown subset of the predictors in $A$ is thought to be potentially irrelevant. For this purpose, we develop multiple shrinkage predictive estimators along with general minimaxity conditions. Finally, we provide an explicit example of a minimax multiple shrinkage predictive estimator based on scaled harmonic priors. (This is joint work with Larry Brown, Feng Liang and Xinyi Xu).

Xin Guo  
Department of IEOR  
University of California Berkeley  

Optimal Order Placement in a Limit Order Book

Recently, various research works showed that deterministic/static trading strategy of slicing big orders over a period of time is optimal to minimize the price impact. These methods may be valid on a macroscopic timescale. However on the millisecond time scale the price is no longer well defined and the state of the order book contains important information. We derive an optimal strategy to place orders in a limit order book that minimizes the expected cost. We show that the optimal strategy is path dependent and depends on key order book statistics.
Ronald Graham  
Department of Computer Science and Engineering  
University of California San Diego

Edge Flipping in Graphs

In this talk I will describe some recent work with Fan Chung on certain Markov processes on graphs. These processes are a special case of random walks on left regular band semigroups, and in the case of graphs, have solutions with a particularly nice structure.

J. Michael Harrison  
Graduate School of Business  
Stanford University

A Dynamic Principal-Agent Model Based on Brownian Motion

Stochastic control is one of the areas in which Larry Shepp has made seminal contributions. In that domain as in others, his specialty has always been explicit solutions, specifically for problems of optimally stopping or optimally controlling Brownian motion. Such Brownian models are increasingly popular in economics, engineering and allied fields, despite their incumbent foundational difficulties, precisely because of the potential for explicit solutions that Larry’s work has revealed. In one instance after another, researchers working with Brownian models have been able to derive clear-cut insights that were unobtainable using conventional models. A striking recent example is a 2008 paper by Yuliy Sannikov (Review of Economic Studies, 75, 957-984), who successfully analyzes a dynamic principal-agent model in which the effects of an agent’s actions are obscured by Brownian noise. In this talk I will describe Sannikov’s model and results, striving to emphasize aspects of greatest interest from a mathematicians viewpoint.
Ioannis Karatzas
INTECH Investment Management LLC
Mathematics Department
Columbia University

A Planar Diffusion with Rank-based Characteristics

Imagine you run two Brownian-like particles on the real line. At any given time, you assign drift $g$ and dispersion $\sigma$ to the laggard; and you assign drift $-h$ and dispersion $\rho$ to the leader. Here $g$, $h$, $\rho$ and $\sigma$ are given nonnegative constants with $\rho^2 + \sigma^2 = 1$ and $g + h > 0$.

Is the martingale problem for the resulting infinitesimal generator

$$L = 1_{\{x_1 > x_2\}} \left( \frac{\rho^2}{2} \frac{\partial^2}{\partial x_1^2} + \frac{\sigma^2}{2} \frac{\partial^2}{\partial x_2^2} - h \frac{\partial}{\partial x_1} + g \frac{\partial}{\partial x_2} \right)$$

$$+ 1_{\{x_1 \leq x_2\}} \left( \frac{\sigma^2}{2} \frac{\partial^2}{\partial x_1^2} + \frac{\rho^2}{2} \frac{\partial^2}{\partial x_2^2} + g \frac{\partial}{\partial x_1} - h \frac{\partial}{\partial x_2} \right)$$

well-posed? If so, what is the probabilistic structure of the resulting two-dimensional diffusion process? What are its transition probabilities? How does it look like when time is reversed?

This construction involves features of Brownian motion with “bang-bang” drift [3], as well as of “skew Brownian motion” [2]. Surprises are in store when one sets up a system of stochastic differential equations for this planar diffusion and tries to decide questions of strength and/or weakness (cf. [1] for a one-dimensional analogue), and when one looks at the time-reversal of the diffusion. I’ll try to explain what we know about all this, then pose a few open questions. (*This is joint work with E. Robert Fernholz and Tomoyuki Ichiba.*)

REFERENCES


Tze Lai  
Department of Statistics  
Stanford University  

Singular Stochastic Control via Optimal Stopping with Applications to Finance and Queueing Networks

David Siegmund  
Department of Statistics  
Stanford University  

Detection of Changepoints

I will discuss problems of change-point detection where the observations are realizations of a random field. After describing the scientific context of these problems, I will suggest possible solutions based on different assumptions about the structure of the random fields and compare the suggested solutions when the assumptions are satisfied and when they are not.

Albert Shiryaev  
Steklov Mathematical Institute of the Russian Academy of Sciences  

Local Time Approach to the Problem of Testing of Statistical Hypotheses for Brownian Motion
J. Michael Steele
Department of Statistics
University of Pennsylvania

Optimal Stopping Problems: Philosophy and Technique

One of the major themes of Shepp’s work is the explicit — and often beautiful — solution of problems of optimal stopping. In later works the martingale method is given the interpretation of a problem in convex optimization. I’ll provide an exposition of some concrete problems with the hope of refining this philosophical point of view. In particular, the martingale method will be compared and contrasted with the direct methods of dynamic programming. One of the examples to be discussed is from joint work with Alessandro Arlotto, Robert Chen, and the master himself.

Srinivasa Varadhan
Department of Mathematics
New York University

Large Deviations for Random Matrices and Random Graphs

Peter Winkler
Department of Mathematics
Dartmouth College

Coordinate Percolation

In the Sheppian tradition of looking for simple models of phase transition that may succumb to analysis, we describe a version of coordinate percolation for which an exact closed solution has been obtained. In coordinate percolation, vertices of a grid live or die based on events associated with the lines that cross there. In contrast to independent percolation, intended as a model for a porous medium, coordinate percolation arises in scheduling problems. We’ll describe several versions of coordinate percolation, including the solved one (with Lizz Moseman of NIST) and several that remain intriguingly open.
Continuous Glucose Monitoring: An Empirical Bayes-Hidden Markov Approach

The continuous glucose monitor (CGM) is a crucial component of modern artificial pancreas technology. A CGM measures the current generated by glucose molecules in the interstitial space (ISIG) and provides nearly continuous estimates of the blood glucose level (BG). Such estimated BG are used by a control algorithm to determine the dosage for an insulin pump, forming a closed-loop system. However, due to biofouling on the CGM sensor and fluctuations within the endocrine system, the CGM is inaccurate and requires frequent calibration by glucose meter (finger stick, FS). This raises the interesting statistical problem of estimating BG based on ISIG and FS data. We consider a hidden Markov model in which the BG is treated as the mean of the FS and the linear relationship between BG and ISIG is described by a non-stationary Markov process depending on the age of the sensor. We derive the updating rule for the Bayes estimator of BG given the transition kernel of the Markov process and develop a nonparametric empirical Bayes estimate of the Markov kernel. Experiments with our methodology demonstrate its advantage over the algorithm of an existing CGM in a large dataset. This talk is based on joint work with Lee Dicker, Larry Shepp, and Tingni Sun.
POSTERS

Estimating the Long-run Effects of Economic and Meteorological Factors on the Domestic Tourism Demand to Galicia (Spain)
Mª Soledad Otero-Giráldez, Marcos Álvarez-Díaz and Manuel González-Gómez
University of Vigo

Forecasting Tourism Demand using Artificial Neural Networks and Genetic Programming
Marcos Álvarez-Díaz, Mª Soledad Otero-Giráldez and Manuel González-Gómez
University of Vigo

Bayesian Inverse Problems with Monte Carlo Forward Models
Ian Langmore
Columbia University

The Price Impact of Order Book Events
Arseniy Kukanov
Columbia University

Functional Generalized Additive Models
Mathew McLean
Cornell University

Sparse Recovery when the Variance is Unknown: a LASSO-type Approach
Stéphane Chrétien and Sébastien Darses
University of Franche Comté and Université de Provence

Small-time Expansions of the Distributions. Densities, and Option Prices of Stochastic Volatility Models with Lévy Jumps
Ruoting Gong
Georgia Institute of Technology
A Stochastic Delay Model for Pricing Debt and Loan Guarantees
Elisabeth Kemajou
Southern Illinois University Carbondale

Optimal Exercise of Real Options: When should you sell your mansion?
Anna Amirdjanova
University of California, Berkeley

Improved Closed-form Prediction Intervals for Binomial and Poisson Distribution
Jie Peng
St. Ambrose University

Learning Distribution over Permutations through Partial Information
Devavrat Shah
Massachusetts Institute of Technology

Determining State Related Changes in Brain Connectivity
Ivor Cribben and Ragnheidur Haraldsdottir
Columbia University
VOLUNTEERS

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